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Special Contribution

Comments on Analysis of a Static Loading Test

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1. INTRODUCTION

The foundation industry and the engineering practice embrace an undesirable disparity of procedures and schedules of performing a static pile loading test. While the maximum applied load, number of load increments, means of providing reaction load, total test duration, extent of instrumentation, etc. will naturally vary depending on the specific circumstances and objectives, there are explicit rules for what to do and what not to do. Regrettably, the rules are more often than not broken. The following illustrates a few by way of presenting and analyzing actual results from an instrumented test.

2. SOIL PROFILE, INSTRUMENTATION, LOADING SCHEDULE, AND BASIC RESULTS

The test pile was a 400-mm diameter, square, precast concrete pile driven to 18 m embedment at a site in Singapore. According to borehole information, summarized in Fig. 1 the profile comprised an upper 5 m thick layer of sandy silt, a hydraulic fill placed on 6 m of soft marine clay followed by sandy silt at 11 m depth and designated as Kallang and Jurong formations, respectively. The SPT N-indices indicated the fill as compact, the clay as very soft to soft, and the sandy silt dense to very dense. The borehole may not have been drilled close to the test pile; as the analysis will suggest, the first 3 m of the profile designated as sandy silt is probably more likely the soft layer, i.e., the Kallang-Jurong boundary could be at 14 m and not at 11 m depth.

The pile instrumentation consisted of a special telltale system called Glostrex (Hanifah and Lee 2006), a system employing Vibrating Wire (VW) extensometers to measure strain between a series of anchor points. Seven anchor points (see Fig. 1A) were installed at 2, 8, 11, 14, 16, 17, and 18 m depths in the test pile and referenced to the pile head a 0.0 m depth; no stick-up. The strain measurements gave compression between the anchor levels and accumulated compressions gave anchor movements to subtract from the pile head movement to provide the movement of the anchor levels and, eventually, of the pile toe. The strain measurement was obtained as a difference between two VW readings and it will, therefore, never quite have the same accuracy as a single VW record. However, it is independent of bending moment, which a VW pair cannot ensure. In effect, the Glostrex system provides strain values at par with the more conventional system of embedded VW gages. The strain value is the average over the anchor length (distance between anchor levels). Usually, the strain value is assumed to be representative for the midpoint of an anchor length. Theoretically, in a uniform soil showing linearly increasing shaft resistance and no toe resistance, the average axial force in the pile converted from the strain readings between the pile head and an anchor level some distance above the pile toe, is the force at $0.7 (= 1/\sqrt{2})$ times the anchor length down from the pile head. However, for anchor lengths further down the pile, the location gets closer to the mid-point of the anchor length (Fellenius 2021). All here used force-averages calculated from strain measured between the anchor levels are plotted at anchor-length mid-point. Note also that the measurement of movement, provided by the Glostrex system is a "companion" measurement of great value for the analysis.

The maximum applied load was 2,500 kN; 2.5 times the intend working load. The test schedule comprised three phases of loading followed by unloading. Phase 1 comprised four approximately equal increments applied about every hour followed by an about 6-hour load-holding before complete unloading. Phase 2 comprised 2 increments of about 8 and 18 minutes load-holding duration, respectively, the second load level being about three times the first, and the load was maintained for about 6 hours. (The twice larger second load increment was, presumably, due to an accidental overshooting of the load increase). The third load increment, about equal to the first, was held for about one hour and the fourth, slightly smaller, was held for 10 hours before unloading. Phase 3 comprised 10 about equal load increments. The first 5 were held for 10 minutes, each of the following 3 were held for about 1 hour, and the 9th was held for about 4 hours. The 10th was held for about 2 hours. The duration of the test from start to finish was 3 days. Fig. 1B shows the graph of the load vs. time test schedule. Obviously, the crew had some difficulty maintaining the pressure in the jack.

Fig. 1C shows the main test response, the pile-head load-movement and the compression of the pile. At the 2,500-kN maximum applied load, the pile head moved about 50+ mm and the compression of the pile was 10 mm. Thus, the pile toe moved 40 mm. Note that each unloading caused the next phase, Phases 2 and 3, to start not from zero as for Phase 1, but from the strain and movement remaining from the preceding test phase.

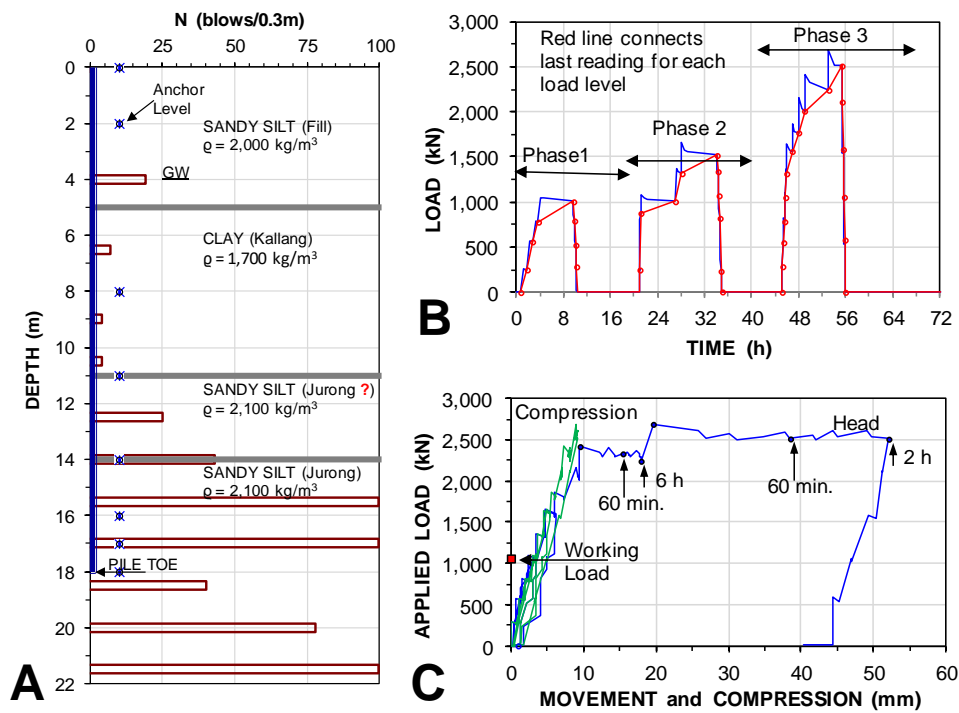


Fig. 1. Soil profile, Anchor levels, Load-time schedule, and Load-movement

Figure 2 shows the strain in the pile remaining at the end of Phase 1 and additional strain remaining at the end of Phases 2 and 3 as well as the total strain induced by the three phases. Such strain is commonly referred to as "residual strain" and the corresponding force is "residual force". It is likely that had an additional phase been introduced after Phase 2 and before the last phase, additional residual strain would have been added to the records. Or, on the other hand, if Phases 1 and 2 had not been part of the schedule, Phase 3, now being the first phase, would have had no such added strain incorporated. Of course, some strain was likely induced during the driving and by set-up following the driving.

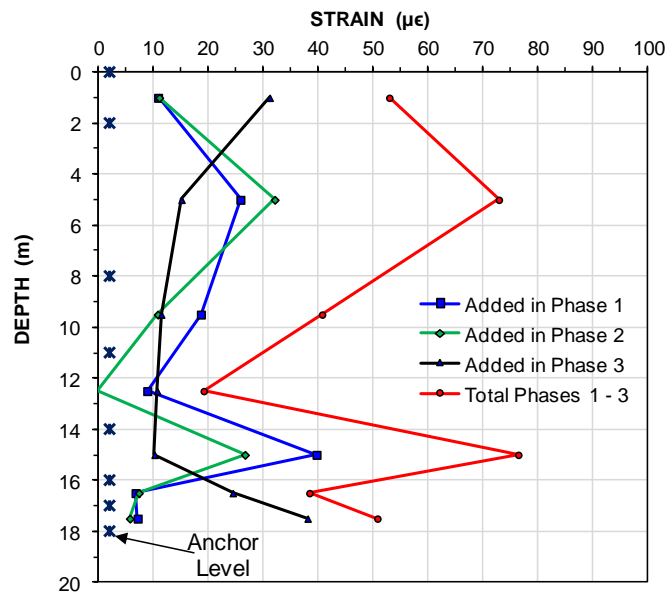


Fig. 2. Strain remaining in the pile after unloading the applied load

In contrast to the subject test, most test schedules are a bit more consistent than the current test in regard to load-holding durations and load levels. Moreover, the applied load levels on actual tests are usually better maintained. However, the main principle is often the same: the tests include a few to several unloading-reloading events or phases—

"cycles" they are not. For years, I have asked colleagues to tell me their rational for specifying a schedule of unloading/reloading and their including uneven load increments, as opposed to scheduling a series of equal increments each held for an equal length of time. The former schedule means spending a not inconsiderable extra amount of money—a client's money or a tax payers' money—a rational for what the extra spending buys or if the spending is warranted is never declared. To date, nobody advocating the former schedule has been able to tell me what is gained by the former over the latter. Of course, when pressured for a reply, many will say that the Code requires it, which to me is the same as the of-old excuse: "the Devil made me do it"! The following analysis will show what unloading-reloading and uneven load-holdings will do to a test interpretation. On a positive note, it will also illustrate how the load-movement results of the Glostrex gages can be used to analyze the response of a pile to applied loads.

3. DATA ANALYSIS

The first analysis step after compiling and organizing the test data is to convert measured strain, ϵ , to axial force, Q . This requires knowing the pile material secant modulus, E_s and the pile cross sectional area, A ; $Q = E_s A \epsilon$. However, for a concrete pile, such as the subject test pile, the E_s -modulus can range from a value ≈ 25 GPa through ≈ 35 GPa, depending on concrete quality and amount of reinforcement in the pile. Moreover, for the subject pile, the nominal area has to be corrected for the central void. Thus, neither E nor A is known with acceptable accuracy. However, the secant stiffness, $E_s A$, can be determined directly from the measurements by, for each record, plotting a graph of the load divided by the strain versus strain. The secant method requires that (1) the zero-force reference of the first reading is accurate, (2) the measured strain is unaffected by locked-in strain, and (3) no or only a minimal amount of shaft resistance develops between the load application and the strain measurement location. This limits the method to a record from a gage located close to the location of applying the load; the pile head for the subject case.

The stiffness of the pile at gage levels away from the load application can alternatively be determined by the tangent stiffness method, which employs plotting the change of applied load (Q) divided by change of strain ($\Delta Q/\Delta \epsilon$) against strain (ϵ) to determine the tangent stiffness, $E_t A$. There is a simple mathematical relation between the tangent stiffness, $E_t A$, and the secant stiffness, $E_s A$. The tangent method is independent of zero shift and residual force. However, it is a differentiation approach and it is therefore very sensitive to data error or inadequate accuracy. The latter means that the applied load, the load that caused the strain at the gage level and the force that caused the change of strain must be kept very stable, because, otherwise, the delay before a load change at the load application point (pile head, usually) is registered as a force change at the gage level would impact the veracity of the interpretation. The tangent method requires that shaft resistance response is plastic, i.e., constant (after an initial movement). If, in contrast, the soil response is strain-hardening, i.e., continues to increase with increasing force, the evaluated stiffness (modulus) increases with strain. If strain-softening, the tendency is instead to decrease with increasing strain. Moreover, the commonly appearing downward slope of the stiffness line (by linear regression), indicating reducing modulus for increasing strain, will be considerably exaggerated by no-plastic shaft resistance. For details, see Fellenius (1989; 2021).

Note, the strict definition of stiffness is a spring constant and it is really $E_s A/L$ and, therefore, " $E_s A$ " should perhaps be called something else, say "stiffness number", which, however, could be confusing. The term "rigidity number" has been used, but "rigidity" is associated with resistance to bending and the context here is axial force, so, I prefer to stay with the term "stiffness".

Fig. 3 shows the secant and tangent stiffness graphs for the subject case records. The secant stiffness method was applied to the strain measured between the pile head and the anchor level at 2 m below the pile head. Linear regression analysis for the values showed that only in Phase 3 was the maximum applied load carried to a load level that would present a secant stiffness development in Phase 3. If maintaining the readings immediately before the start of Phase 1 as the zero reference to Phase 3 records, a secant stiffness, $E_s A$, of 4.14 GN was obtained. The total strains remaining after each unloading of Phases 2 and 3, were 7 and 13 $\mu\epsilon$, respectively, for the gage length between the pile head and 2.0 m depth. The graph shows the secant modulus calculated after subtracting those values. The Phase 3 linear regression then showed $E_s A$ to be 4.50 GN, instead, which is an about 8 % larger value. N.B., the subtraction of the strains is an uncertain action as the data also include the effect of hysteresis.

The bottom graph shows the tangent stiffness plot for the Phase 3 (Phase 1 and 2 did not engage the pile sufficiently). Note that the values from the last two load levels are affected by the erratic loads applied in the test (c.f. Fig. 2). The pile has a uniform cross section and therefore, the secant method stiffness obtained for the uppermost gage level is representative for all the strain records. Thus, applying the tangent method (bottom graph) to the measured strains would seem to be redundant. However, the plot is still worthwhile because it adds reference to the representativeness of the records. The tangent method applied to the records indicated that the stiffness is 4.5 GN, which is the same as that shown by the secant method after correcting for zero level. As mentioned, the tangent method is independent of the zero-strain

reference uncertainty. Therefore, the tangent method result confirms that, for this test, the residual strain at the start of the test should be subtracted from the records before calculating and plotting the secant values. Moreover, no significant strain-hardening appears to be present.

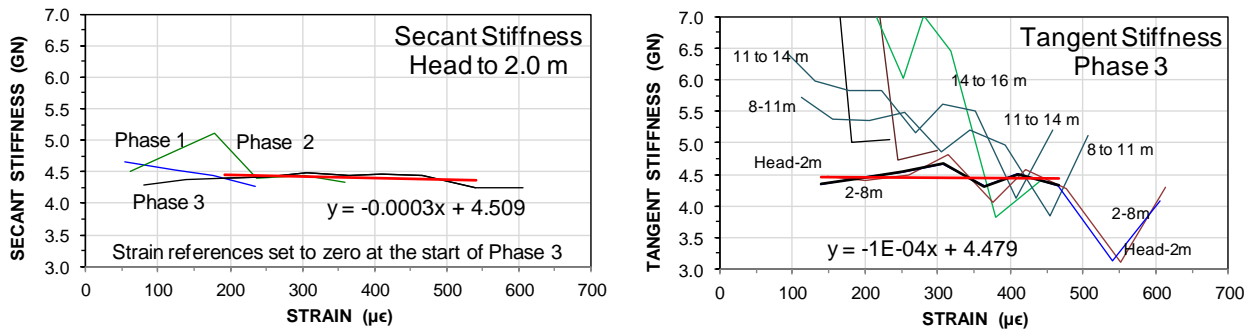


Fig. 3. Secant and tangent stiffness versus strain

The so-established secant stiffness, $E_s A = 4.5 \text{ GN}$, was applied to convert the measured strains to axial force in the in the pile. Fig. 4 shows the force distributions (blue line) converted from the about 60-minute measurement of each of the 10 load levels of Phase 3, L3-1 through L3-10, and for the maximum load applied in Phases 1 and 2, L2-4 and L2-5. Values from the anchor length between 2 and 8 m depths appeared to be erratic and were not plotted. My next step was to choose the L3-7 distribution as a suitable target for a back-calculation of distribution of axial force that gave a fit to measured and calculated values. The fit is indicated using red circles connected with a red line. The analysis presents the distribution of mobilized unit shaft resistance. It is convenient to express this in relation to an effective stress distribution, employing the soil profile parameters. This determines a distribution of a ratio, called β -coefficient, between the effective stress and force reduction from the pile head to the pile toe and the β -coefficients that gave the fit are listed to the right. N.B., the calculation represents the shaft resistance mobilized for the Target Load and it is not intended to represent an ultimate shaft resistance.

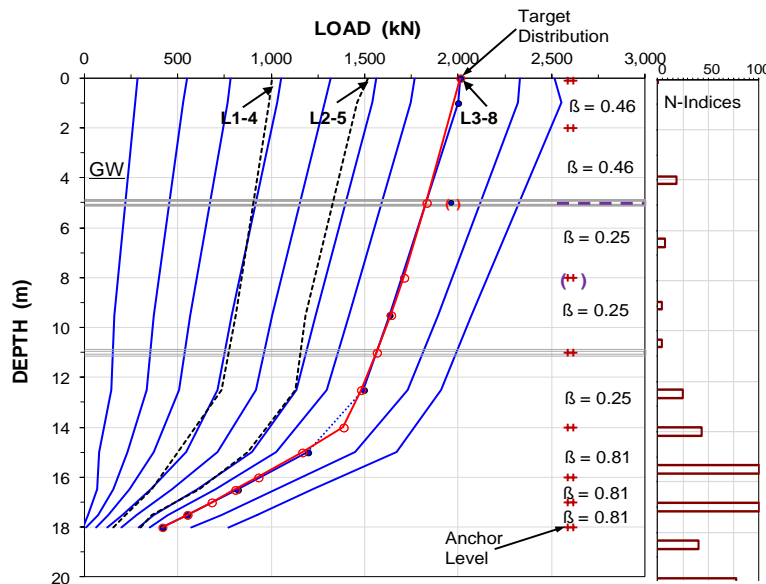


Fig. 4. The strain records converted to force and plotted at the mid-point of each anchor length

The figure includes the N-index diagram as reference to the soil profile and the back-calculation suggests that the softer middle zone extends about 3 m deeper than given in the borehole log. From the load levels L3-5 (1,316 kN) through L3-8 (2,010 kN), the curves are parallel, which indicates plastic shaft resistance. The last two curves are slightly less steep, which would indicate strain hardening, but it is more likely due to the erratic load-holding for the load levels.

For comparison, the analysis of the Phase 3 records was also carried out without adjusting for the strain induced by the two preceding phases. With regard to the β -coefficients, the difference between the results were slight, as indicated in Table 1, but for the middle portion of the pile. The more noticeable difference was with the toe resistance obtained by linear extrapolation of the curve over the lowest 3 m length of the pile. The adjusted toe resistance for the target

distribution was 416 kN, whereas the toe resistance for the unadjusted records was 480 kN. The unadjusted analysis overestimated the target toe force by 15 %.

Table 1. Distribution of β -coefficients with and without correction of strain from preceding test phases

Depth (m)	0.0 - 5.0	5.0 - 14.0	14.0 - 18.0		
Corrected	0.46	0.25	0.81		
Depth (m)	0.0 - 5.0	5.0 - 11.0	11.0 - 15.0	15.0 - 15.0	16.0 - 18.0
Not corrected	0.50	0.26	0.24	1.10	0.80

The next step was to fit calculated and measured force-movement curves as indicated in Fig. 5. The circles show the measured force-movements and the curves show the simulated and fitted curves. The simulations employed t-z functions for the shaft resistance and a q-z function for the toe resistance. The t-z function was separated on the lower soil layer, "Jurong formation" and a layer combining the "Kallang formation" and fill, thus, employing two t-z functions and the toe resistance q-z function for the analysis as shown in Fig. 6. The fit is good for the first eight load levels, while it is much less good for the last two, which is considered due to the erratic load holding for these last two steps of the test. Of course, a fit to the large-movement records could have been forced, but it would have resulted in a poor fit for the beginning of the test. A similarly good fit for the unadjusted test records could also be produced, but it required a series of different t-z functions and the solutions are therefore less credible. The q-z function was obtained with a function coefficient, θ , of 0.30, which is a very low value indicative of presence of a residual toe force. A function coefficient in the range of 0.6 through 0.8 is more common for virgin conditions. (The simulation calculations and fits were made with the UniPile software; Goudreault and Fellenius 2014).

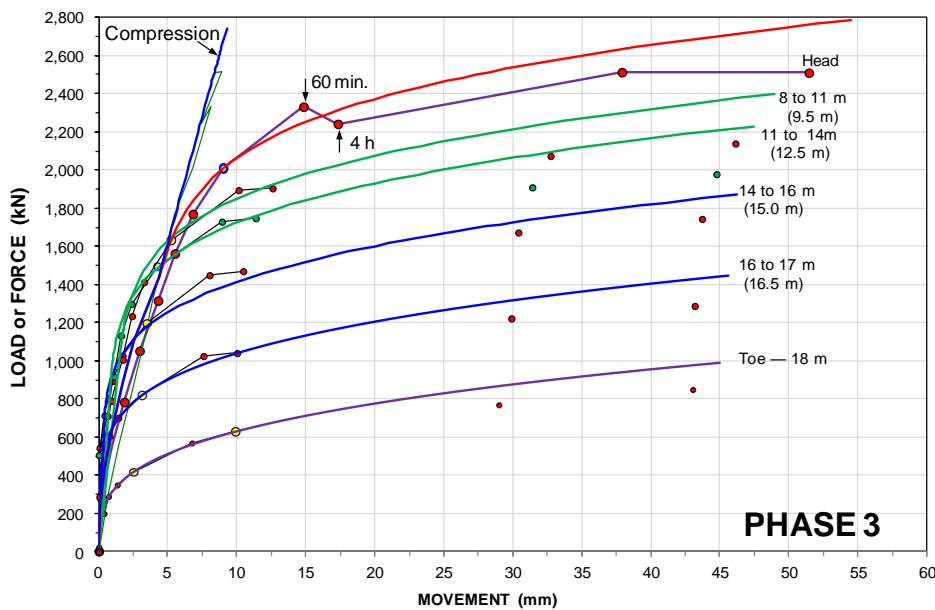


Fig. 5. Load-movement curves for the pile-head and gage levels

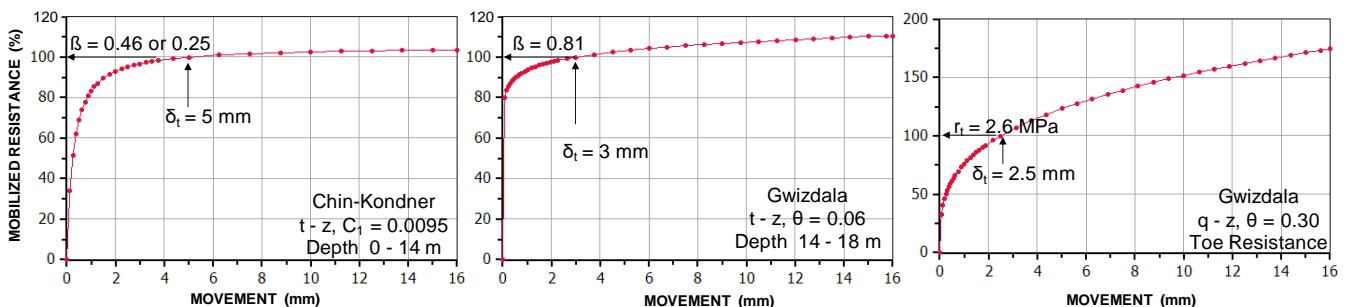


Fig. 6. The t-z and q-z functions that gave the fit

Having established the load-movement response of the pile would seem to provide all information needed for use in the piled foundation design as described in my previous newsletter item. However, there is still the issue of the locked-in force in the pile affecting the test. Residual force may or may not have been introduced due to driving and set-up, though some certainly would have. Moreover, the attempt to remove the effect of the Phases 1 and 2 may not have been sufficient to change the "before-test" conditions for Phase 3 to those for Phase 1. Regardless, it is very likely that the Phase 3 test was affected by presence of residual force developed from the pile head down along most of the pile length.

An analysis (N.B., for the Target Distribution of axial force in the pile) to establish the probable amount of residual force and its distribution relies on the assumption that the negative direction of shear force is equal to the positive direction, that is, the shear resistance unaffected by residual force is represented by a β -coefficient distribution not smaller than half that back-calculated from the test records. However, unless measured directly, it is not known how far down this condition is valid, that is, at what depth does the negative direction change to positive direction of shear and what is the length of this transition. Also not known is the magnitude of unit shaft resistance along the pile below the transition zone and the magnitude of the final toe resistance. The boundary conditions for the distribution are (1) that the depth to the start of the transition is at the pile toe, which also optimizes the final toe resistance, (2) the final toe resistance cannot be smaller than that determined in the test, (3) that for whatever toe resistance chosen as the "correct" final toe resistance, the slope of the positive shaft resistance rising from that value cannot be steeper than the slope evaluated from the test condition, and (4) the length of the transition zone is unknown; it depends on the relative movement between the pile and the soil in developing the residual conditions—it is usually several pile diameters in length. The actual distribution will be somewhere within the boundaries and establishing it requires applying suitable judgment. Fig. 7 compares a probable distribution after correcting the measured Phase 3 distribution for residual force likely resulting from the driving and set-up following the driving combined with additional residual force due to Phases 1 and 2 not accounted for by re-zeroing the strain and movement records. The analysis suggests that the residual pile-toe force was about 200 kN and the true toe resistance mobilized by the 2,000 Target Load was about 600 kN. Separating out and knowing resistance contribution from the pile toe is always complex. When knowing the pile toe force is important, as for the subject test, performing a bidirectional test is always preferable—the axial force in the pile established in a bidirectional test is true and unaffected by residual force.

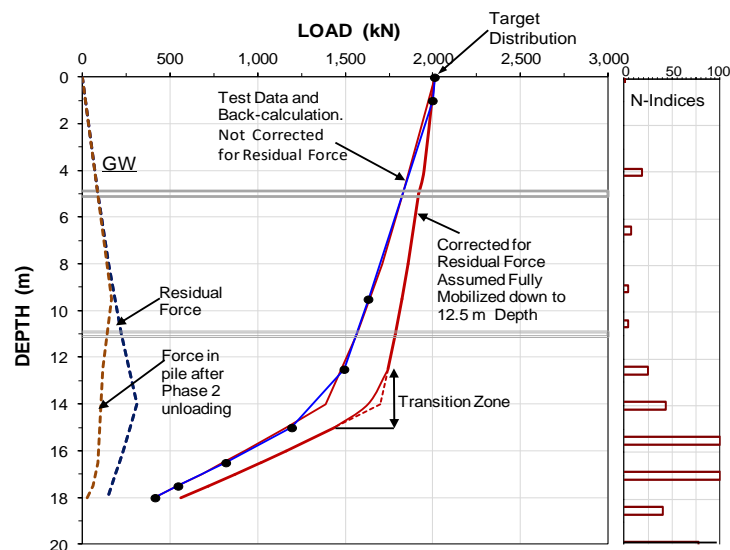


Fig. 7. Distribution of target force before and after correction of potential residual force

The back-calculation of the measured distribution of axial force gave the distribution of shaft resistance along the pile expressed in values of unit shaft resistance, r_s , as well as correlated to effective stress, σ'_z , by a β -coefficient ($\beta = \sigma'_z/r_s$). The relation Fig. 8 shows these distributions for the uncorrected and corrected conditions of residual force. The unit shaft resistance is a function of effective stress, usually considered a proportionally relation, and, therefore, it increases with depth. However, conventionally, the unit shaft resistance is treated as a constant, stress-independent number, which is false. In contrast, the β -coefficient is usually constant within a uniform soil and it is therefore a much simpler parameter to use in an analysis. As long as the effective stress is brought into the analysis, the two approaches are identical. Note, the linear distributions shown in the figure are based on assuming an immediate transition from negative to positive direction of residual shaft shear, disregarding that it actually occurs over a transition length.

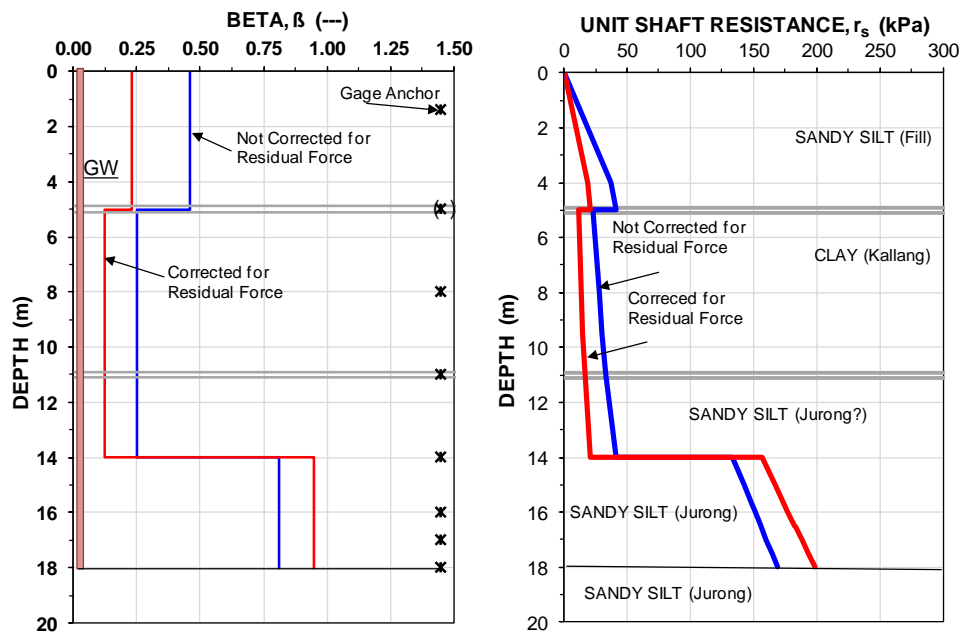


Fig. 8. Distribution of β -coefficient and unit shaft resistance before and after correction for residual force

The presence of residual force will not have had any significant effect on the shaft resistance response in terms of force-movement, i.e., the t - z functions for the virgin conditions would be approximately unchanged from those determined for the test conditions. However, the virgin toe resistance will show a considerable softer initial response—the residual toe force will appear as a prestressing force and the initial part of the force-movement curve will then be in reloading, appreciably stiffer than in virgin loading. A re-analysis of the load-movement response of the pile under assumption of shaft resistances equal to those determined from the corrected force distribution, applying the same t - z functions as used for the simulations shown in Fig. 5, but with the t - z function changed to $\theta = 0.60$ and toe resistance (developed at the 2,000-kN Target Load) of 400 kN, results in the pile-head load-movement and a pile-toe force-movement curves shown in Fig. 9. Of course, the calculated response depends on the assumptions made in regard to the residual force present in the pile at the static test and the calculated test curves for the assumed virgin test could either be closer to and more deviating from the measured. However, even with a more optimistic set of assumptions, the difference between the pile-head response as determined in the static loading test and that corrected for the assumed presence of residual force is substantial and will affect the results of using the results for design decisions. In assuming a very conservative distribution of residual force, a similarly corrected analysis can be used for a load-movement and settlement analysis that aims to decide if the design for a piled foundation supported on piles similar to the test pile is suitable.

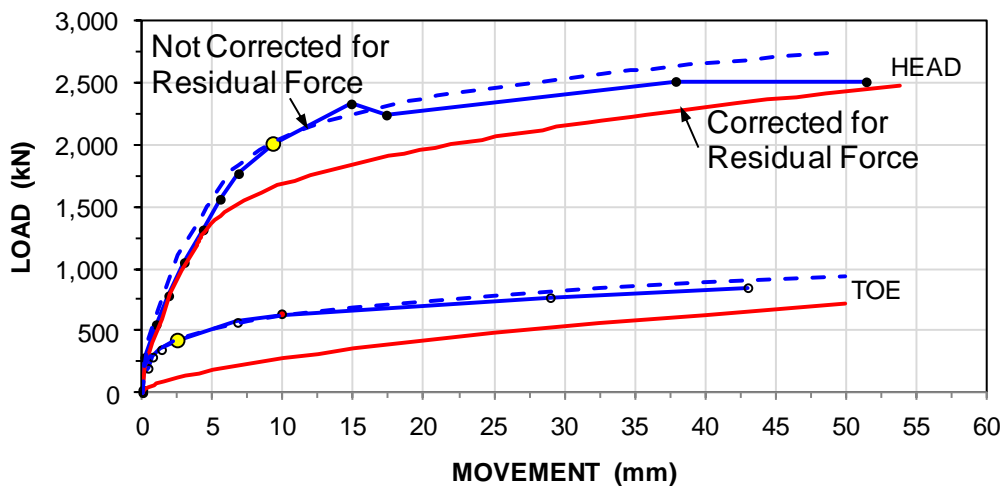


Fig. 9. Measured and corrected pile-head load-movement and pile-toe force-movement curves

4. CONCLUSIONS

Several observations result from the study of the records and analysis of the of the records of the static loading test on the instrumented precast concrete pile.

1. The soil exploration was intermittent and very limited. It would have been useful to see samples from sampling made at closer separations as well as seeing information from laboratory tests, in particular, compressibility variation, e.g., from the oedometer tests on Shelby tubes. The results from a CPTU sounding or two would have been very supporting to the analysis.
2. The primary problem with obtaining an accurate analysis lies with the testing schedule incorporating unloading and reloading events, which adversely affected the analysis of the main test, Phase 3.
3. Because of a delay before a change of applied load fully affects the strain measured down the pile, the demonstrated difficulty in holding the applied load constant affected the ability to correlate measured strain to axial force.
4. The fact that the testing schedule comprised uneven magnitudes of load increments and varying load holding durations introduced additional imprecision for the test analysis. My attempt to correct for this by subtracting the strain remaining in the pile before starting the main test phase is only partially effective as the loading and unloading were active at different levels of residual strain in the pile.
5. It is likely that residual force resulted from the driving and set up after the driving. However, the imposition of the two unloading and reloading events before the main test made it difficult to entirely assess the amount of residual force and its distribution.
6. For additional comments on aspects that can affect the interpretation of the results of a static loading test, see Fellenius and Nguyen (2019).
7. A several-day test duration infers considerable costs that instead of adding value to a test reduces it. Performing instead a test with a series of equal load increments (say about $20\pm$) applied at equal times, (rarely longer than 15 minutes) and with each load level well maintained and held for equal duration would add values and save costs. The savings could be used to pay for the extra soil sampling and analysis and a cone penetrometer test for "free" enhancement of the test value.
8. Notwithstanding the problems due to the unloading-reloading and load-holding variations imposed by the project designers, the analysis of the test results provided insight in the pile response that can be extrapolated to use of piles of different size and length and/or site conditions, e.g., effect of fills and excavations, for foundations on single piles and pile groups contained in the project.
9. The Glostrex telltale-type instrumentation proved to function very well, adding significant value to the loading test.

5. ACKNOWLEDGEMENT

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