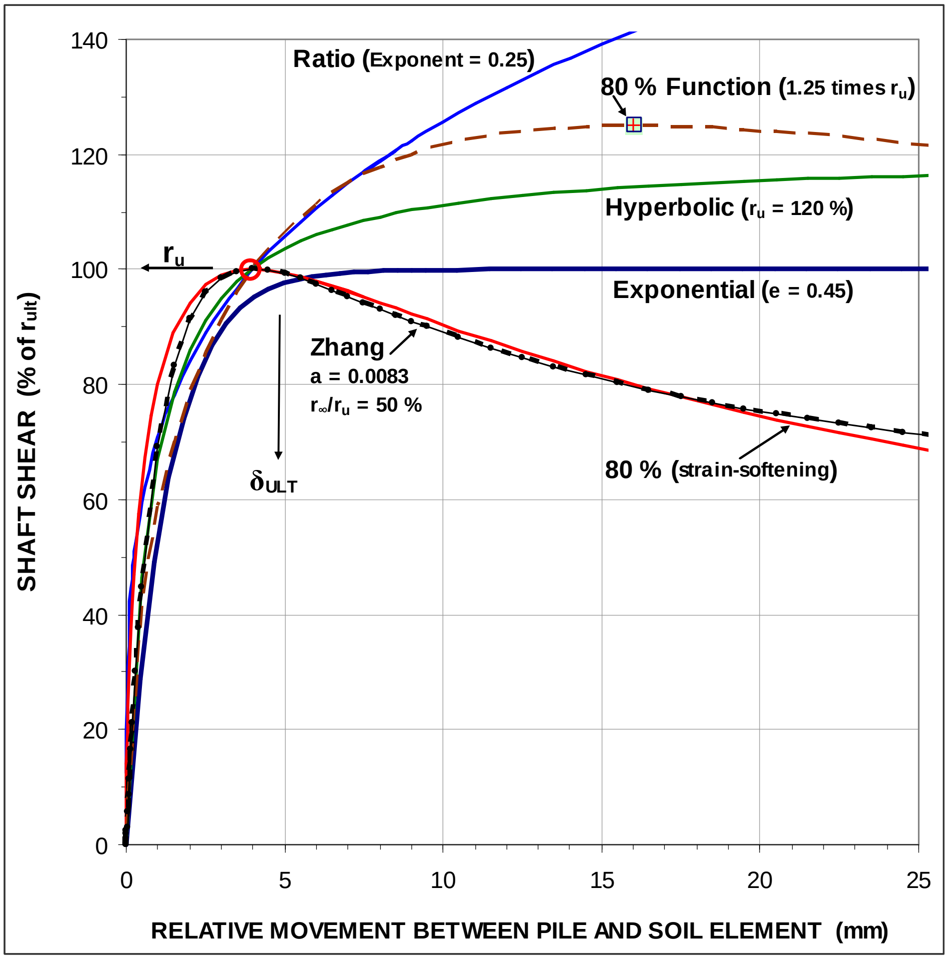

Suggested t-z and q-z functions for load-movement responsef

Fellenius, B.H., 2013. Simplified non-linear approach for single pile settlement analysis. Discussion. Canadian Geotechnical Journal, 50(6) 685-687.

Discussion on Zhang Q.Q. and Zhang, Z.M., 2012. Simplified non-linear approach for single pile settlement analysis. Canadian Geotechnical Journal, 49(11) 1256-1266

by Bengt H. Fellenius, Dr.Tech., P.Eng.
Canadian Geotechnical Journal, 50(6) 685-687

The authors have proposed an interesting t-z function for use in modeling strain-softening pile shaft response of a pile subjected to a static loading test. A t-z function describes the relation between the amount of stress (or load) necessary to generate movement of a soil element (or pile head) in a static loading test. The authors' function fits in well with existing functions: the Ratio Function, the 80-% Function, the Hyperbolic Function, and the Exponential Function, summarized below.

The **Ratio Function** is defined by Eq. D1.

Eq. D1 $$r = r_u \left(\frac{\delta}{\delta_u}\right)^b$$

where
- r = resistance variable
- r_u = ultimate resistance
- δ = movement variable
- δ_u = movement mobilized at r_u
- b = an exponent ranging from a small value through unity

The **80-% Function** is defined by Eqs. D2a through D2c.

Eq. D2a $$r = \frac{\sqrt{\delta}}{C_1\delta + C_2}$$

Eq. D2b $$C_1 = \frac{1}{2r_u\sqrt{\delta_u}}$$

Eq. D2c $$C_2 = \frac{\sqrt{\delta_u}}{2r_u}$$

where
- r = resistance variable
- δ = movement variable
- r_u = ultimate resistance
- δ_u = movement at ultimate resistance, r_u
- C_1 = the slope of the straight line in the √δ/q versus movement (δ) diagram
- C_2 = ordinate intercept of the straight line in the $\sqrt{\delta}/r$ versus movement (δ) diagram

The **Hyperbolic Function** is defined by Eq. D3.

Eq. D3 $$r = \frac{\delta}{C_1\delta + C_2}$$

where
- r = resistance variable
- δ = movement variable
- C_1 = $1/r_u$ (also the slope of the line in a r/δ vs. movement (δ) diagram,
- C_2 = ordinate intercept
- r_u = ultimate resistance

The **Exponential Function** is defined by Eq. D4.

Eq. D4 $$r = r_u(1 - e^{-b\delta})$$

where
- r = resistance variable
- δ = movement variable
- b = coefficient
- e = base of the natural log. (2.718)

The authors' function, here called the **"Zhang" Function**, can be rewritten as shown in Eqs. D5a through D5e.

Eq. D5a $$r = \frac{\delta(a + c\delta)}{(a + b\delta)^2}$$

Eq. 5b $$r_u = \frac{1}{4(b - c)}$$

Eq. D5c $$\delta_u = \frac{a}{b-2c}$$

Eq. D5d $$b = \frac{1}{2r_u} - \frac{a}{\delta_u}$$

Eq. D5e $$c = \frac{1}{4r_u} - \frac{a}{\delta_u}$$

Eq. D5f $$\frac{r_u}{r_\infty} = \frac{c}{b^2}$$

where	r	=	resistance variable
	r_u	=	ultimate resistance
	r_∞	=	resistance at infinite movement
	δ	=	movement variable
	δ_u	=	movement mobilized at r_u
	a	=	"independent" coefficient
	b and *c*	=	"dependent" coefficients

The t-z functions can be employed to fit a calculated stress-movement record to the response measured in a static loading test. It is best achieved by first establishing from the measured curve the ultimate resistance, r_u, and the movement for this resistance, δ_u, employing suitable definitions and judgment. As each of the five t-z curves rely on a single additional parameter, a simple trial-and-error approach will achieve the best fit between the calculated and measured curves. Fitting by the 80-% and Hyperbolic functions can be speeded up by determining the respective C_1 and C_2 parameters from a linear regression over a suitably selected range of measured r-δ values.

The five functions are illustrated in Figure D1 comprising load-movement curves calculated for an assumed ultimate resistance, r_u, of 100 %, occurring at a movement, δ_u, of 4 mm. The 80-% function is always strain-softening after the ultimate resistance. The figure shows that the Zhang function for a strain-softening to 50 % of r_u at large movement is practically equal to the curve calculated by the 80-% function. However, the Zhang function allows for the post-peak softening to take different shapes, whereas the 80-% function has a fixed shape (once the values of r_u and δ_u are selected).

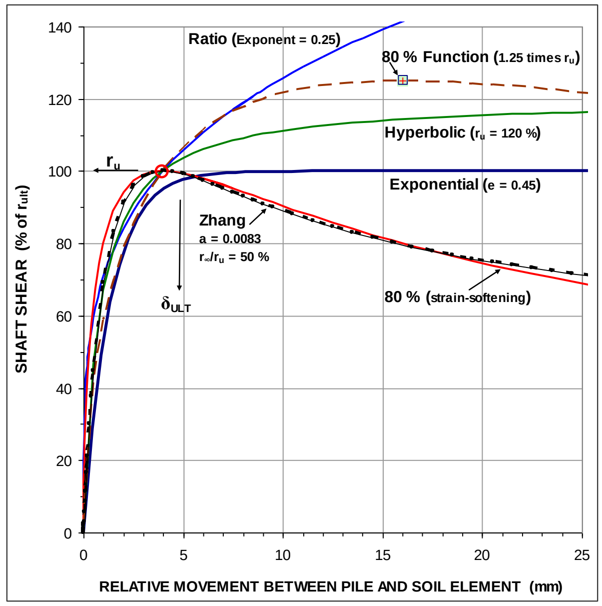

Fig. D1 Compilation of t-z curves for a common r_u = 100 % and δ_u = 4 mm

The independent coefficient, "a" controls the degree of strain-softening after the peak (100 %) shear resistance is mobilized. It can range from 0.0000 for no strain-softening, through 0.0100 for loss of all (100 %) of shear resistance at large ("infinite") movement. For δ_u = 4 mm, "a" = 0.0083 corresponds to a 50-% reduction at large ("infinite") movement and a = 0.0100 results in 100 % reduction.

In an actual case, different soil layers will have different stress-movement curves. Some will be strain-softening, as in the authors' case, other layers will be strain-hardening. While the figure shows the curves for a common point, r_u and δ_u, the curves can be made to be quite different before and after the common point. Therefore, it is always possible to fit one or more of the functions to a given measured stress-movement curve.

The definition of the 80-% function is the requirement that the stress-movement curve must also go through a point that has a stress equal to 80 % of the chosen ultimate resistance, r_u, developing at a movement that is equal to 25 % of δ_u. Thus, the function can be used to model also the stress-movement for a case where an "ultimate resistance", r_u, is assumed to occur prior to a "peak resistance", if the peak value and its movement are assumed to be $1.25r_u$ and $4\delta_u$, respectively.

The authors also proposed that the pile toe stress-movement (or load-movement) should be modeled by a bi-linear curve. This recognizes that a pile toe does not show a failure mode, but the resistance always increases with movement. However, numerous full-scale static loading tests with measurements of toe stress versus toe movement have shown that the pile toe response is always curved for both the initial portion and the large movement portion, and the pile toe q-z response is usually similar to the Ratio function. Figure D2 shows a typical Ratio function stress-movement curve from start to large movement (150 mm) and a bi-linear relation fitted to the response with the "kink" at 25-mm movement.

Figure D3 shows the same records plotted with the maximum movement reduced to 25 mm. The stress scale is the same as in Figure D2. While the assumed bi-linear response could be adjusted to a reasonable fit between the start and up to the 25-mm movement with a "kink" at about a movement of 5 or 10 mm, the new set would not fit the response beyond 25-mm. To fit beyond 25 mm would require that the bi-linear approach be changed to tri-linear. And if the fit would be to the range from the start to 5 mm (the authors' toe movement reference), as well as to several ranges beyond 5 mm, a multi-linear approach would be necessary. Indeed, the bi-linear approach is an unnecessary simplification, because as the Ratio function (which best represents the pile toe response) can just as easily as the bi-linear response be coded into a computer software for analysis of the response.

The information presented for the authors' parametric study of an 800 mm diameter, 20 m long pile in a soil with a uniform shaft resistance of 50 KPa does not include the toe resistance. However, the 50 KPa unit shaft resistance value results in a total shaft resistance of 2,500 KN and, as the authors' Figures 7 and 8 show a final resistance of 8,000 KN at about 30 mm pile head movement, the pile toe resistance at the applied load of 8,000 KN would be 5,500 KN. This toe resistance is stated to have developed at the maximum pile head movement of about 30 mm. As the pile compression for the 8,000 KN pile head load is about 25 mm, the pile toe movement at the 8,000 KN load is about 5 mm. However, a toe resistance of 5,500 KN mobilized at such small toe

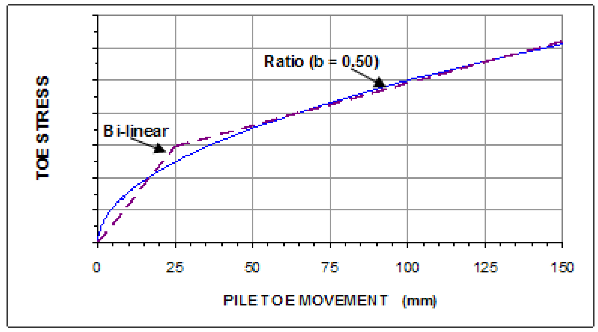

Fig. D2 Toe stress-movement by bi-linear modeling and by the Ratio method to 150 mm

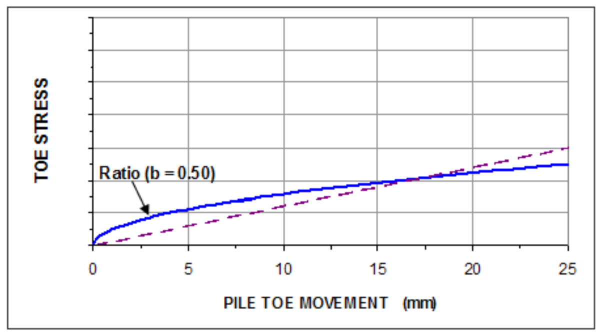

Fig. D3 Toe stress-movement by bi-linear modeling and by the Ratio method to 25 mm

movement does not correlate well to a shaft resistance of only 50 KPa immediately above the pile toe (presumably the soils above and below the pile toe are similar). It is not likely that the shaft resistance would be constant with depth, however. It would normally be smaller near the ground surface and larger at depth and, thus, correlate better to the toe resistance. The authors' assumption of constant shaft resistance is an additional unnecessary simplification.

The shaft resistance for a 20 m long pile in homogenous soil, whether it is sand or clay would be responding in accordance to the effective stress. An effective stress analysis employing a constant value of effective stress beta-coefficient would have been a more realistic assumption than the authors' assumption of constant unit shaft resistance. The 2,500 KN total shaft resistance correlates to a beta-coefficient of about 0.5, a large value in most non-residual soils. When the results from a static loading test appear to show a load distribution answering to a constant unit shaft resistance, this is often a consequence of residual load in the pile. Piles, also bored piles, are usually affected by locked-in load ("residual load"), which needs to be considered in every evaluation of test results.

I realize that the parametric study is not an actual case and used to demonstrate the versatility of the authors' computer program. However, inasmuch as the paper presents conclusions pertaining to engineering practice, a more "real" case would in my opinion have served better for demonstrating the results of the authors' methods and analyses.