

Pile Driving Formulas

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Dynamic formulas were commonly used in the early 1900s to estimate driven pile capacity, and many comparisons were then made with static loading tests. A look at the past is often helpful in understanding what should (or should not) be done in the present.

ASCE formed a Committee in 1930 to review dynamic formulas. After a decade long study, the "Committee on Pile Driving Formulae and Tests" produced two reports in May 1941 and sparked a remarkable series of 28 discussions in the ASCE Proceedings by Terzaghi, Casagrande, Peck, Tschebotarioff, and Proctor to mention only a few (Likins et al. 2012).

Considering the current search by some agencies to find a better dynamic formula, primarily to increase the LRFD resistance factor to make pile designs more economical, it is prudent to review what these geotechnical "giants" thought about pile driving formulas, when they were widely used and "the only trick in the book".

Formulas evolve

First consider the then prevailing conditions. Pile sizes were typically twelve inches or smaller. Wood piles were common. Drop hammers or single-acting steam hammers dominated. The diesel hammer, common today, had not yet been introduced to America. Hydraulic hammers had not been invented. Soil mechanics was still in its infancy. There were no accepted standards for conducting static loading tests or interpreting the resulting data.

The first documented formula use in America was by Major John Stanton in 1851 for timber piling to support Fort Delaware. The 6,000 piles, driven to a sand layer, took three years to install using a 2,000 pound drop weight.

In December 1888, Arthur Mellen Wellington published a formula in Engineering News. This 'Engineering News' formula, designed for drop hammers and timber piles (Chellis, 1951), was widely used for decades. Wellington was a realist, however, and stated "In so very uncertain a matter, it is wrong in principle to start from high ultimates, which are certainly unsafe as a unit, and allow foolish men to deceive themselves with the notion that they are being cautious, when they divide it by three or four, but are really running great risks".





A. Hiley published a formula in 1925 that was more "complete" trying to account for various "losses". Additional formulas arose after 1940, such as the Gates formula promoted by the U.S. Bureau of Public Roads (now Federal Highway Administration). R.D. Chellis lists more than 30 different formulas in his 1951 textbook *Pile Foundations*, but noted that actual factors of safety "may vary from as low as 1/2 to as high as 16 or more".

Since there were many formulas already in use, the primary goal of the ASCE Committee was to determine which formula to recommend. Some defended the more complete or extensive formulas, while others essentially said, "Why bother with complexity?" and suggested that the simpler formulas were just as accurate, or just as unreliable.

Terzaghi writes in his 1942 discussion, "*The use of the (dynamic) formula in the design of pile(d) foundations is unsound on both economical and technical grounds*".

Terzaghi also wrote in the preface of his textbook "*Theoretical Soil Mechanics*", published in 1943: "*In spite of their obvious deficiencies and their unreliability the pile formulas still enjoy a great popularity among practicing engineers, because the use of these formulas reduces the design of pile foundations to a very simple pro-*

cedure. The price one pays for this artificial simplification is very high. In some cases, the factor of safety of foundations designed on the basis of the results obtained by means of pile formulas is excessive and, in other cases, significant settlements have been experienced"

SUMMARY of the 1941-1942 DISCUSSIONS

The following summary of the 1941-1942 Excerpted discussion quotes (in italics) illustrate the position of each discussor. A copy of the complete original discussion can be obtained from the second author.

Discussion terms "dynamic analysis", "dynamic test", or similar, refer to the now common term "dynamic formula" since modern dynamic testing (e.g. ASTM D 4945) with a Pile Driving Analyzer and signal matching CAPWAP software, as well as "wave equation analysis" (e.g., GRLWEAP) were still decades into the future. The terms used of "load testing" or "loading tests" or simply "tests" similarly refer to "static loading tests" (e.g. ASTM D1143).

September 1941 issue

Greulich (Carnegie-Illinois Steel Co.) "*The use of formulas, without a thorough knowledge of all factors at the site that might influence pile behavior and without check tests, may lead to serious error — either by an unsafe or a very*



uneconomical and extravagant design. The writer would be opposed to the publication of any formula unless the dangers and pitfalls of its use are made very clear.”

Engel (Modjeski and Masters) discusses “set-up” for timber piles in Louisiana. “Any dynamic formula would assign totally different allowable loads to these piles before and after their rest periods, and it would seem the wisest course, therefore, to use no dynamic formula for friction piles.”

Watson (Assistant Professor, Duke University) “Report B recommends nothing except (static) load testing of piles to failure. The writer wishes further to deplore the moribund attitude that prompted other members of the Committee to prepare ‘Report A – Pile Formulas’. Although they may fervently wish to have a formula for the ready solution of their problems, they should not ask the Society to fulfill their prayer by promulgating a Committee formula, unless they can prove their case in court.”

October 1941 issue

Chellis (Stone and Webster) – “Engineering News formula is not the general answer to the problem”. Chellis used the Hiley formula and “found its use very practicable. The older formulas give widely varying results with different types of piles and hammers, entirely out of reason.” He declares “non-validity of a dynamic formula when driving into cohesive soils” and cautions “the formula is very sensitive at small penetrations”.

White (President, Spencer, White and Prentis) “The proposed formula has the failings of all previous pile formulas – it can only give the value at the time of driving (if it can even do that) and not 24 h later. Moreover, the writer’s firm has repeatedly underpinned structures that should not have suffered from settlements – were the Engineering News formula reliable. Furthermore, it would be a calamity for the Society to lend its authority to the promulgation of any pile driving formula as yet described.”

Mason (Bridge Engineer, State of Nebraska) “Pile driving formulas are a necessity.”

Proctor (Moran, Proctor, Freeman, and Mueser) “the large immediate value of this Report is in its warning to designing engineers as to the fallacies of pile formulas and the weaknesses of pile tests.”

Paaswell (Spencer and Ross) “When one persists in the quest for a pile formula, one ignores or merely gives lip service to the science of soil mechanics. Soil mechanics and the pile formula are essentially incompatible.”

Woolf (Albert Kahn, Inc.) “Terzaghi, in 1925, gave his soil mechanics lectures discussing the validity of pile driving formulas. It was then emphasized that one should be careful with formulas, particularly the Engineering News formula. Report B definitely proposes that no formula be used, but that (static) load tests be resorted to. This proposal is a difficult one to accept, but basically it is sound and correct.”

November 1941 issue

Evans (Bethlehem Steel) “protests against the development or use of any ‘pile driving formula’ as such. It is misleading and unsafe to seek a magic combination of terms, in a formula, that will fit any and all cases regardless, and which is supposed to indicate just what load the pile will support.”

Atwood (Consulting Engineer) “It would seem that there are no formulas of general or even local value unless they are treated with good judgment and corroborated by many tests. If that is true, why try to use a formula? It would seem, with the knowledge now available, that the best the Committee could do would be to make some very general statements as to the unsafeness of using formulas, and the necessity for (static) testing and the exercise of judgment.”

Burmister (Columbia University) “In view of the limitations of any pile driving formula and of the uncertainties involved in the successful application to the installation of pile foundations in any given situation, it is believed that Report B (static tests) represents the better practice. Once a formula has been printed, it takes on a more or less authoritative character, and the assumption on which it is based and the limitations in its use tend to be forgotten or overlooked. It seems doubtful that any consistent relationship can exist that will be of general application for different types of soil.”

Belcher (United Engineers and Constructors, Inc.) “Neither Report A nor Report B places any reliance on the Engineering News formula. The attempt to introduce a new formula (Report A) is of very doubtful value, as it is based on the same fundamental data that invalidate the Engineering News formula.”

Williams (President, Lehigh University) “A formula having complicated refinements is not consistent with the nature of the problem.”

Krynine (Yale University): “Of the two reports, A and B, the latter (promoting static tests) is preferable”. He then discusses the sensitivity of the Hiley formula to the numerous constants and factors, and states:

“All these questions should be clarified in the Manual, if, unfortunately, the Hiley formula is recommended for the general use. The writer sincerely hopes, however, that this will not happen.”

December 1941 issue

Dames and Moore (Dames and Moore Inc.) *“Dynamic formulas that are restricted to drop hammers will be of limited value at best. The scatter of data is so wide that the only conclusion possible is that the dynamic formulas are unreliable and, in most cases, are likely to lead to unnecessarily expensive construction costs.”*

Upson (Raymond Concrete Pile Company) *“The simplest possible formulas and information should be advocated. Each formula should be accompanied by a clear statement of its usefulness and limitations. It is the writer’s apprehension that the presentation of complicated formula such as Eq. 9 (simplified Hiley formula), requiring so many assumptions, may well lead the uninitiated engineer astray.”*

Tschebotarioff (Princeton University) *“Any dynamic pile driving formula is nothing more than a yardstick to help the engineer secure reasonably safe and uniform results over the entire job. The use of a complicated formula is not recommended since such formulas have no greater claim to accuracy than the more simple ones.”*

Feld (Consulting Engineer) *“The true difference between the two reports is whether the design of piles shall be based on a dynamic test (formula) as checked by the static test, or on the static test alone. Personally, the writer would prefer to have the Manual covering pile driving formulas include a definite formula for*

granular soils, a definite formula for plastic soils, and a definite formula for such conditions as end-bearing piles in which no lateral restraint or resistance is to be expected. Dynamic (formula) are useless in plastic soil.”



January 1942 issue

Mohr (M. ASCE) *“After studying the formula derived in Report A and ‘worrying’ through Mr. Hiley’s published work, upon which analysis the proposed formulas are based, it is the writer’s firm conviction that their inclusion in the proposed Manual would be a grave mistake. Answers obtained by its use are no more consistent and logical than those obtained by the use of other formulas. Its only obvious advantage to those who wish to be critical of present formulas is the great number of unknowns to which a series of values may be applied until an answer satisfactory to the interested party is finally reached.”*

Cummings (Raymond Pile Driving Company) *“In the writer’s opinion, the publication of Report A in a Manual of Practice would be a serious mistake. There are only five basic types of dynamic pile driving formulas in use at the present time and all of them can be represented by the formula $Wh = R_s + Q$ in which Q represents all the energy losses that occur during impact. For many years, engineers*

have been making all kinds of assumptions as to what should and what should not be included in Q . The profusion of pile driving formulas that can be found in engineering literature is simply the result of these assumptions. There is available a very considerable amount of pile driving data from which it is possible to determine indicated bearing capacities by means of a number of dynamic formulas and then to compare these computed results with the actual bearing capacity determined by a load test to failure. When such data are tabulated, it is always seen that some of the computed results are several hundred per cent above or below the actual test results.”

February 1942 issue

Terzaghi (Harvard University) *“The defects of the pile driving formulas are either due to disregarding variable and vital factors (Engineering News formula), or they are due to the inadequate evaluation of the influence of these factors on the effect of the blow of the hammer (general equation and its derivatives). The formulas of both groups share the defect that they disregard the energy transmission through the pile by elastic waves. The degree of reliability of a formula can be measured by the range of scattering of the ratio between computed and real values about the statistical average. In spite of the waste of material and labor involved in an average factor of safety of 4, an occasional failure is inevitable. Whoever uses the formula is in exactly the same position as the man who tries his luck on a gambling machine. He is at the mercy of the laws of probability”.*

Peck (Chicago Engineering Dept., later University of Illinois) contributes *“Report A carries the implication that pile driving formulas give the results that have some relationship to the ultimate bearing capacity of piles. The validity of some or any of these formulas can be determined only by comparison of ultimate loads found by loading tests and by the formulas. On the basis of the data in Table 2, it can be demonstrated by a purely statistical approach that the chances of guessing the bearing capacity of a pile are better than of computing it by a pile driving formula... The statistical study indicates that the use of a pile driving formula is merely a somewhat inferior method of permitting the laws of chance to operate in the determination of pile capacity”.*

Casagrande (Harvard University) “*The question of ‘pile formulas’ has without doubt been the most controversial issue in the field of civil engineering for a hundred years. The question of how to treat the chapter on pile formulas is indeed a difficult one, particularly in view of the desired standard expressed in the first paragraph of the Manual manuscript: ‘This manual ... endeavors to enunciate sound principles which are based on established facts, and to avoid stating rules or giving formulas which might lead to its unintelligent use. Rigorous adherence to this desirable goal would eliminate all pile formulas, since they are certainly not based on ‘established facts’; nor can one say that one can recommend any formula and feel reasonably sure that it might not lead to its unintelligent use’.*”

March 1942 issue

Dunham (Yale University) “*A formula which depends upon various and variable coefficients, whose values are subject to guessing and change without notice, is confusing and deluding. Everyone agrees that the results obtained from such a formula are not correct but, if they are reasonably so and moderately conservative, one may as well arrive at the results simply rather than through devious mathematical procedures whose greater value is probably psychological rather than real’.*”

May 1941 issue

Closure by Admiral Bakenhus (U.S. Navy, Ret.) “*‘Pile formulas’ is the one subject upon which the Committee has reached no definite stated conclusions. Tests (e.g. static) cost relatively little in extensive operations, but may be relatively large and even out of the question with the smaller project. At its best, the pile driving formulas are merely an empirical method for predicting the safe bearing load for a single pile. Experience has shown that there is no determinable fixed relation between the safe bearing value of a pile and the factors used in the formula. It is, therefore, a dangerous proceeding for an engineer to design or build a piled foundation solely on the*

information obtained by the usual test of measuring penetration per blow, height of fall, and weight of hammer.’” He addresses many of the discussers’ points, but notes they do “*not suggest what the engineer in the Midwest prairies should do when he has a total of perhaps twelve piles under some bridge foundation, and when neither funds nor time permit (static) load tests or soil analysis. This is one of the difficult problems before the Committee’.*” Today, of course, this quandary is resolved by means of dynamic monitoring of the piles.



DISCUSSION

The discussers from the early 1940’s show a clear consensus about the unreliability, unscientific basis, uncertain outcome, and risk for using dynamic formulas. A weakness of any formula is the actual hammer performance of any individual hammer is variable — and unknown. Modern dynamic testing with the PDA clearly shows actual measured energy transfer may vary by a factor of two among supposedly identical hammer models and types. It is no wonder that the discussers state that they had experienced poor correlation of dynamic formulas to static tests.

Several discussers note that formulas should be restricted to cohesionless soil applications. Chellis (1951) states “*a formula can apply only in the case of cohesionless strata, such as sand, gravel or permeable fill*”. Yet today this intended restriction is ignored. Current thought equates the long-term set-up gain in cohesive soils to the dynamic viscosity of the soil during installation. This false assumption may be correlated to give the mean formula result similar to the mean static test result, but on any individual site the coefficient of variation may result in gross errors, as explained by Rausche et al. (2004).



Later research further confirms these failings of formulas. Olson and Flaate (1967) studied 93 piles driven in sands with static loading tests. They suggested different “adjusted forms” for the Gates formula for different pile types, which includes individual “constants” (for each pile type) for multiplying the energy term (that differ by almost a factor of two between wood and steel piles; this likely improvement is not used today so results suffer). An argument could be made for using a similar approach with regard to different soil types, but, then, what would be the appropriate formula for layered soils? Combinations of “adjusted forms” for different pile types in different soil types would end in mass confusion.

Lawton et al (1986) made an extensive literature study, including results of nine published correlation studies by others, and a survey of most of the State Departments of Transportation. They found that “*the ENR formula, either in its original form or more often in a modified version, is by far the most popular dynamic formula used.*” This is alarming since 8 of the 9 correlation studies “*found the ENR and modified ENR formulas to be among the worst.*” Lawton also found “*All investigators were consistent with regard to wave equation methods. A wave equation analysis of static pile capacity was consistently equal to or better than the best formula predictions, despite old versions of wave equation computer programs being used in many studies in which input information was not always accurate.*” They reasonably surmise even better correlations with newer wave equation programs and accurate input information.

Today, the typical pile, pile driving hammer, and pile capacities greatly exceed (by an order of magnitude or more) the capacities in the databases used to develop the formulas. Hannigan (2006) notes for the ‘Engineering News’ formula that with a modern data base “*The fact that 12% of the data base has a factor of safety of 1.0 or less is also significant.*”

A.E. Cummings in his 1942 discussion was prophetic in his assessment “*As a matter of fact, the only new concept that has been introduced into pile driving formula in the past fifty years is the theory of the longitudinal impact of long elastic rods. This theory is not new, as it was developed by St. Venant (1857) and Boussinesq (1885) many years ago. The application of the theory to pile dynamics was first suggested by D.V. Isaacs (1931) and the British Building Research Board in 1938 and demonstrated the fact that the behavior of full size piles under actual field conditions can be predicted with considerable accuracy by means of this theory. The theory is concerned with the question of stress transmission through the pile and, unfortunately, it involves some rather difficult mathematics. However, there is a considerable amount of field evidence available which shows that the stress transmission characteristics of a pile are of great importance not only in determining its behavior during driving but also with respect to its subsequent ability to carry static load. This method of investigating the phenomena of pile driving dynamics is one that deserves the careful attention of all engineers engaged in pile driving work. It is a new and promising field for investigation* (authors’ emphasis)”. Fortunately, this method has been further developed in the wave equation (initially developed about that time by Mr. Cummings’ associate at the Raymond Pile Driving Company, Mr. E.A.L. Smith).

The Wave Equation analysis can correctly model the pile and hammer, and the resulting wave transmission. Wave equation soil models account for pile viscosity and soil layers. The largest unknown is then the actual hammer performance and energy transfer.

An even better use of the stress wave propagation theory mentioned by Cummings is now common in dynamic pile testing. Since the mid-1970s, dynamic pile testing and signal matching analyses clearly estimated the capacity more accurately at the time of testing, either during installation or during restrike. (Hannigan 2006).

If measured results from dynamic testing are considered in a “refined wave equation analysis” (Rausche et al 2009) the resulting bearing graph is even more reliable. Since the more scientific wave equation analysis is readily available, user-friendly, and takes little more time to run an analysis than to make the formula calculation, the question then is why are formulas still in use? Even more incredibly, there are still funded studies for development of new formulas.

J.G. Mason (Bridge Engineer, State of Nebraska) stated in his 1941 discussion “*Pile driving formulas are a necessity.*” From a historical perspective, this was reasonable in 1941. Engineers then needed some way to evaluate when to stop driving the pile. Some today might start with a dynamic formula to preliminarily select the hammer for a certain pile capacity. But it is bewildering to encounter current project specifications that evaluate pile capacity by means of only a dynamic formula. On larger projects, a static loading test is always a good idea. On any project, prudence would suggest a dynamic pile test, or at least a wave equation analysis. Compared with reliance only on formulas, better engineering, including either static or dynamic testing, almost always results in a more economic design at significantly reduced risk.

This brief review of the extensive discussion comments is presented to produce more realistic expectations of what can or cannot be achieved by a dynamic formula. Hopefully this summary of the 1941-1942 discussions will not just provide information of historical interest, but also will encourage more modern engineering of piled foundations.

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