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From Strain Measurements to Load in an Instrumented Pile

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Introduction

More and more, our profession is realizing that a conventional static loading test on a pile provides limited information. While the load-movement measured at the pile head does establish the capacity of the pile (per the user's preferred definition), it gives no quantitative information on the load-transfer mechanism (magnitude of the toe resistance and the distribution of shaft resistance). Yet, this information is what the designer often needs in order to complete a safe and economical design. Therefore, more and more frequently, the conventional test arrangement is expanded to include instrumentation to obtain the required information can be obtained. Normally, the instrumentation consists of strain gages placed at selected levels to determine the load at that location for each load applied to the pile head. The gages are used to measure strain and this article provides guidelines for how to convert the strain to load.

Aspects to consider

In arranging for instrumentation of a pile, several aspects must be considered. The gages must be placed in the correct location in the pile cross section to eliminate influence of bending moment. If the gages are installed in a concrete pile, a key point is how to ensure that the gages survive their installation—a gage finds encountering a vibrator a most traumatic experience, for example. We need the assistance of specialists for this work. The survival of gages and cables during the installation of the pile is no less important and this requires the knowledge and interested participation and collaboration of the piling contractor, or, more precisely, his field crew.

Once the gages have survived the pile manufacture and installation—or most of the gages, a certain redundancy is advised—the test can proceed and all should be well. That is, provided we have ensured the participation of a specialist having experience in arranging the data acquisition system and the recording of the readings. Then, however, the geotechnical engineer often relaxes in the false security of having all these knowledgeable friends to rely on. He fails to realize that the reason for why the friends do not interfere with the testing programme and testing method is not that they trust the geotechnical engineer's superior knowledge, but because advising on the programme and method is not their mandate.

The information obtained from a static loading test on an instrumented pile can easily be distorted by unloading events, uneven load-level durations, and/or uneven magnitude of load increments. Therefore, a static test for determining load transfer should be carried through in one continuous direction of movement and load followed by unloading without disruptions. (Unloading and re-loading cycles are best saved to a repeat test after the completion of the undisturbed first test).

So, once all the thoughts, know-how, planning, and hands-on have gone into the testing and the test data are secured, the rest is straightforward, is it not? No, this is where the fun starts. These notes will address how to turn strain into load, a detail that often is overlooked in the data reduction and evaluation of the test results.

Converting to load using the elastic modulus

Strain gages are usually vibrating wire gages. The gages provide values of strain, not load, which difference many think is trivial. Load is just strain multiplied by the cross sectional area of the pile and the elastic modulus, right?

The modulus of a steel is known quite accurately, but the modulus of concrete is not. The latter can vary within a wide range, and common relations for its calculation, such as the relation between the modulus and the cylinder strength, are not reliable enough. A steel pile is only an all-steel pile in driving—during the test it is often a concrete-filled steel pipe. The modulus to use in determining the load is the combined value of the steel and concrete moduli. By the way, in calculating the concrete modulus in a concrete-filled steel pipe, would you choose the unconfined or the confined?

Well, the question of what modulus value to use is simple, one would think. Just place a gage level near the pile head where the load in the pile is the same as the load applied to the pile head, and let the data calibrate themselves, as it were, to find the concrete modulus. However, in contrast to the elastic modulus of steel, the elastic modulus of concrete is not a constant, but a function of the imposed load, or better, the imposed strain. Over the large stress range imposed during a static loading test, the difference between the initial and the final moduli for the pile material can be substantial. This is because the load-movement relationship (stress-strain, rather) of the tested pile, taken as a free-standing column, is not a straight line. Approximating the curve to a straight line may introduce significant error in the load evaluation from the strain measurement. However, the stress-strain curve can with sufficient accuracy be assumed to follow a second-degree line: $y = ax^2 + bx + c$, where y is stress and x is strain (Fellenius, 1989). The trick is to determine the constants a and b (the constant c is zero).

The approach builds on the fact that the stress, y, can be taken as equal to the secant modulus multiplied by the strain. This is achieved by way of first determining the tangent modulus, and then using it to determine the secant modulus. The following presents the mathematics of the method.

Mathematics of the method

For a pile taken as a free-standing column (case of no shaft resistance), the tangent modulus of the composite material is a straight line sloping from a larger tangent modulus to a smaller. Every measured strain value can be converted to stress via its corresponding strain-dependent secant modulus.

The equation for the tangent modulus line is:

(1) $$M_t = \left(\frac{d\sigma}{d\varepsilon}\right) = A\varepsilon + B$$

which can be integrated to:

(2) $$\sigma = \left(\frac{A}{2}\right)\varepsilon^2 + B\varepsilon$$ (the integration constant can be assumed equal to 0)

However,

(3) $$\sigma = E_s\,\varepsilon$$

Therefore,

(4) $$E_s = 0.5\,A\,\varepsilon + B$$

where
M_t = tangent modulus of composite pile material
E_s = secant modulus of composite pile material
σ = stress (load divided by cross section area)
$d\sigma$ = $(\sigma_{n+1} - \sigma_1)$ = change of stress from one load increment to the next
A = slope of the tangent modulus line

ε = measured strain

$d\varepsilon$ = $(\varepsilon_{n+1} - \varepsilon_1)$ = change of strain from one load increment to the next

B = y-intercept of the tangent modulus line (i.e., initial tangent modulus)

With knowledge of the strain-dependent, composite, secant modulus relation, the measured strain values are converted to the stress in the pile at the gage location. The load at the gage is then obtained by multiplying the stress by the pile cross sectional area.

Procedure

When data reduction is completed, the evaluation of the test data starts by plotting the tangent modulus versus strain for each load increment (the values of change of stress divided by change of strain are plotted versus the measured strain). For a gage located near the pile head (in particular, if above the ground surface, the modulus calculated for each increment is unaffected by shaft resistance and the calculated tangent modulus is the actual modulus. For gages located further down the pile, the first load increments are substantially reduced by shaft resistance along the pile above the gage location. Therefore the load change at the gage is smaller than the increment of load. Initially, therefore, the tangent modulus values calculated from the full load increment divided by the measured strain will be large. However, as the shaft resistance is being mobilized down the pile, the strain increments become larger and the calculated modulus values become smaller. When all shaft resistance above a gage location is mobilized, the calculated modulus values for the subsequent increases in load at that gage location are the composite tangent modulus values of the pile cross section.

For a gage located down the pile, shaft resistance above the gage will make the tangent modulus line plot below the modulus line for an equivalent free-standing column—giving the line a translation to the left. The larger the shaft resistance, the lower the line. However, the slope of the line is unaffected by the amount of shaft resistance above the gage location. The lowering of the line is not normally significant. For a pile affected by residual load, strains will exist in the pile before the start of the test. Such strains will result in a raising of the line—a translation to the right—offsetting the shaft resistance effect.

It is a good rule, therefore, always to determine the tangent modulus line by placing one or two gage levels near the pile head where the strain is unaffected by shaft resistance. An additional reason for having a reference gage level located at or above the ground surface is that such a placement will also eliminate any influence from strain-softening of the shaft resistance If the shaft resistance exhibits strain-softening, the calculated modulus values will become smaller, and infer a steeper slope than the true slope of the modulus line. If the softening is not gradual, but suddenly reducing to a more or less constant post-peak value, a kink or a spike will appear in the diagram.

Example

To illustrate the approach, the results of a static loading test on a 20 m long Monotube pile will be used. The pile is a thin-wall steel pipe pile, tapered over the lowest 7.6 m length. (For complete information on the test, see Fellenius et al., 2000).

The soil consisted of compact sand. Vibrating wire strain gages were placed at seven levels, with Gage Level 1 at the ground surface. Gage Levels 2 through 5 were placed at depths of about 2, 4, 9, and 12 m. Gage Level 6 was placed in the middle of the tapered portion of the pile, and Gage Level 7 was placed at the pile toe.

The loads and associated measured strains are presented in Fig. 1. Because the load-strain curves of gages 1, 2, and 3 are very similar, it is obvious that not much shaft resistance developed above the Gage Level 3.

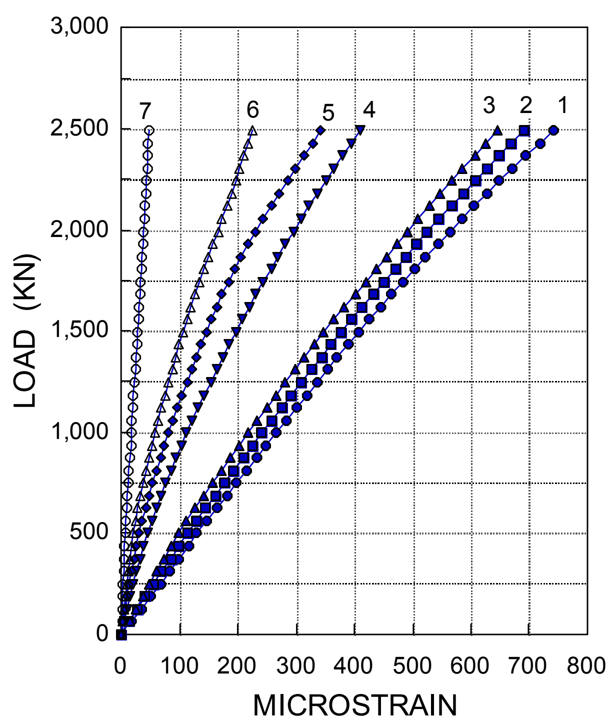

Fig. 1 Strain measured at Gage Levels 1 through 7

Fig. 2 shows that tangent modulus values for the five gages placed in the straight upper length of the pile, Gages Levels 1 through 5. The values converge to a straight line represented by the "Best Fit Line".

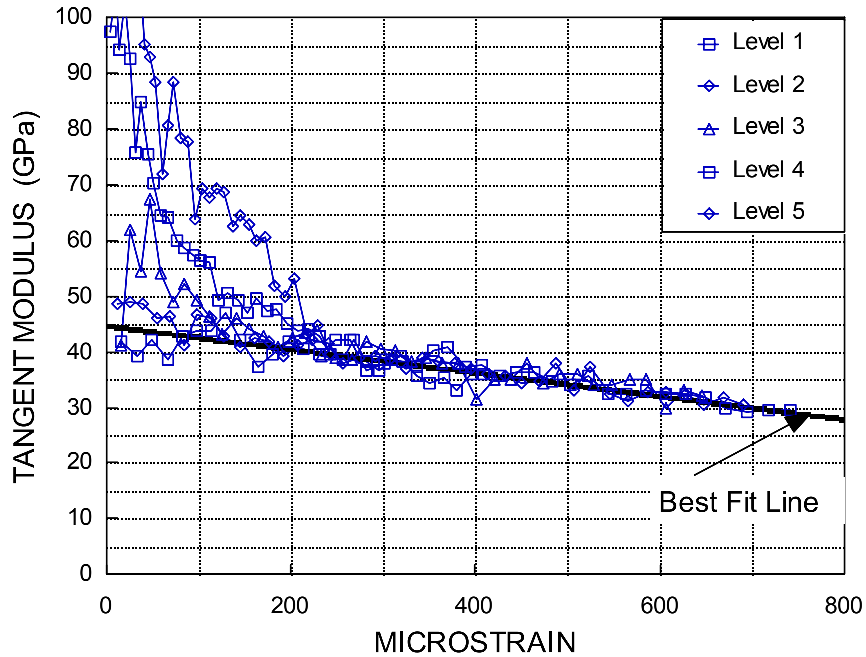

Fig. 2 Tangent Modulus Diagram

Linear regression of the slope of the tangent-modulus line indicates that the initial tangent modulus is 44.8 GPa (the constant "B" in Eqs. 1 through 4). The slope of the line (coefficient "A" in Eqs. 1 through 4) is -0.021 GPa per microstrain ($\mu\varepsilon$). The resulting tangent moduli are 40.5 GPa, 36.3 GPa, 32.0 GPa, and 27.7 GPa at strain values of 200 $\mu\varepsilon$, 400 $\mu\varepsilon$, 600 $\mu\varepsilon$, and 800 $\mu\varepsilon$, respectively. The corresponding secant moduli are 42.7 GPa, 40.5 GPa, 38.4 GPa, and 36.2 GPa, respectively.

To illustrate the importance of establishing the strain dependency of the modulus: at the applied load of 2,400 KN, Gage Level 3 located at a depth of 5 m registered a strain of 625 $\mu\varepsilon$. At the same load, Gage Level 5 at a depth of 12 m registered a strain of 271 $\mu\varepsilon$. The strain values correspond to stress levels of 21.9 MPa and 9.6 MPa, respectively. If the 36 GPa average constant modulus had been used, the stress levels would have become 22.5 MPa and 7.8 MPa and the shaft resistance acting between the two levels would have been determined with an about 10 percent to 20 percent error.

The pile cross sectional area as well as the proportion of concrete and steel change in the tapered length of the pile. The load-strain relation must be corrected for the changes before the loads can be calculated from the measured strains. This is simple to do when realizing that the tangent modulus relation (the "Best Fit Line") is composed of the area-weighted steel and concrete moduli. Conventional calculation using the known steel modulus provides the value of the concrete tangent modulus. The so-determined concrete modulus is then used as input to a calculation of the combined modulus for the composite cross sections at the locations of Gage Levels 6 and 7, respectively, in the tapered pile portion.

Fig 3 presents the strain gage readings converted to load, and plotted against depth to show the load distribution in the pile as evaluated from the measurements of strain used with Eq. 4. The figure presents the distribution of the loads actually applied to the pile in the test. Note, however, that the strain values measured in the static loading test do not include the strain in the pile that existed before the start of the test due to residual load. Where residual loads exist, the values of applied load must be adjusted for the residual loads before the true load distribution is established.

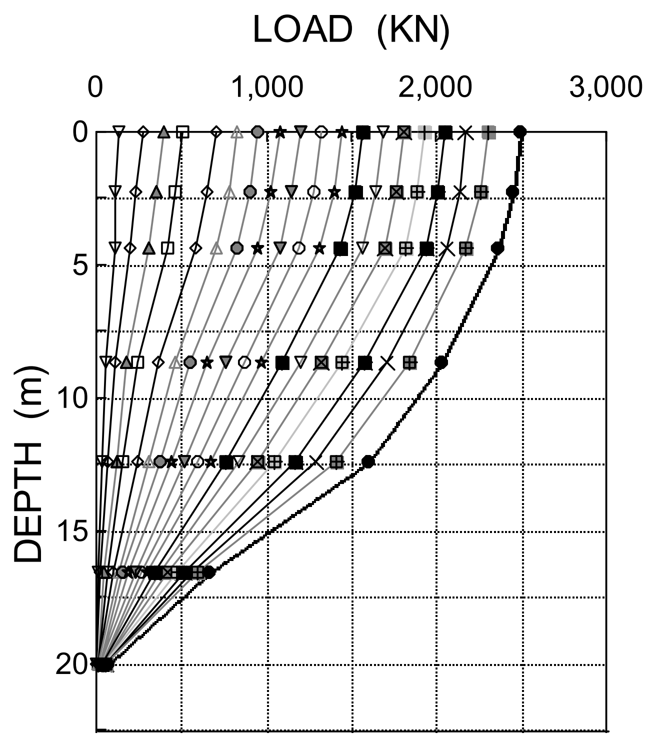

Fig. 3 Load distribution for each load applied to the pile head

Summary

When determining the load distribution in an instrumented pile subjected to a static loading test, one usually assumes that the loads are linearly proportional to the measured strains and multiplies the strains with a constant—the elastic modulus. However, only the modulus of steel is constant. The modulus of a concrete can vary within a wide range and is also a function of the imposed load. Over the large stress range imposed during a static loading test, the difference between the initial and the final tangent moduli for the pile material can be substantial. While the stress-strain relation is non-linear (i.e., curved), in contrast, the tangent modulus of the composite material is a straight line. The line can be determined and used to establish the expression for the secant elastic modulus line. Every measured strain value can therefore be converted to stress and load via its corresponding strain-dependent secant modulus.

For a gage located near the pile head (in particular, if above the ground surface, the tangent modulus calculated for each increment is unaffected by shaft resistance and it is the true modulus (the load increment divided by the measured strain). For gages located further down the pile, the first load increments are substantially reduced by shaft resistance along the pile above the gage location. Initially, therefore, the tangent modulus values will be large. However, as the shaft resistance is being mobilized down the pile, the strain increments become larger and the calculated modulus values become smaller. When all shaft resistance above a gage level is mobilized, the calculated modulus values for the subsequent increases in load at that gage location are the tangent modulus values of the pile cross section.

Results are presented from a static loading test on a pile equipped with vibrating wire strain gages at seven levels. The measured strains were used to plot the tangent modulus values for the gages. The modulus values converged to a straight line showing the secant moduli to reduce from about 40 GPa at the initial loads to about 28 GPa toward the end of the test. Neglecting the strain-dependency of the modulus and using a constant (an average) modulus value would have introduced errors of about 10 percent to 20 percent in shaft resistance determined from the measurements.

References

Fellenius, B. H., 1989. Tangent modulus of piles determined from strain data. The American Society of Civil Engineers, ASCE, Geotechnical Engineering Division, 1989 Foundation Congress, Edited by F. H. Kulhawy, Vol. 1, pp. 500 - 510.

Fellenius, B. H., Brusey, W. G., and Pepe, F., 2000. Soil set-up, variable concrete modulus, and residual load for tapered instrumented piles in sand. American Society of Civil Engineers, ASCE, Specialty Conference on Performance Confirmation of Constructed Geotechnical Facilities, University of Massachusetts, Amherst, April 9 - 12, 2000, 16 p. (***This paper presents details on the continued analysis of the example. A copy is available for downloading from [www.Fellenius.net], File "204 Tapered Piles in Sand.pdf).***