

Bearing Capacity of Footings and Piles—A Delusion?

DFI Annual Meeting, October 14 – 16, 1999, Dearborn, Michigan

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Summary

The bearing capacity equation—“the triple N formula”—is now about 50 years old. It was developed from theory of plastic resistance of metal to a cone being pressed into the surface of the metal, superpositioned by resistance due to shear in an interparticulate medium (soil; note, we are not dealing with metallurgy), and tempered by work necessary to move the footing into the medium. The equation (and its coefficients) was “confirmed” by tests on small footings placed at or near the surface of the soil. The equation is a basic tenet of every geotechnical textbook, manual, and code. Over the years, innumerable variations of the N-coefficients and adjustments to the equation have been published.

The main assumption behind the bearing capacity equation is that a shear plane develops between two essentially otherwise unaffected bodies: the main soil body and a body comprised of the footing with some soil. Moreover, the notion is that a model footing placed at or near the surface of a soil would behave proportionally or similarly to a full-scale footing, or to even a same size footing (pile toe) placed at some depth in the ground. This is not in agreement with factual observations. It is suggested that the behavior of a small or large diameter footing as well as a pile toe is governed by compression of the soil below the footing or pile toe and not by “bearing capacity”.

Arguments are developed with reference to steady state principles and shown to be supported by results from full-scale measurements. It is proposed that design based on a factor-of-safety applied to bearing capacity or on the more modern factored resistance in a limit-state-design are based on a faulty models of behavior and, therefore, may result in less than perfect designs—some may not be reliable and some overly conservative and costly. Settlement analysis should be given a greater role in foundation design; for footings as well as for piles.

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Introduction

Every university course in basic foundation design teaches the calculation of bearing capacity of footings and piles. Usually, the very same course also teaches how to calculate settlement, and to apply the principles to the calculation of settlement of a footing. Very few courses teach how to calculate the settlement of a pile foundation, however.

Simply expressed, students are taught that the bearing capacity is a function of the pertinent shear strength of the soil (undrained shear strength, or cohesion intercept and friction, $\tan \phi$). In school, the shear strength to use is provided by the professor, the text book, or the problem text. In the real world, the undrained shear strength may be known or values to use are provided in the soils investigation, but the friction angle is usually assumed.

For **footings**, the geotechnical engineering practice regularly calculates the bearing capacity from input of assumed shear strength values and a series of relations that depend on these values directly and indirectly. The capacity is then divided by a factor of safety, normally ranging from 2.5 through 4 to obtain the allowable load or stress.

For **piles**, the capacity of the pile toe is assumed to follow a bearing capacity formula (static analysis). However, it is generally thought that the capacity of a pile is so difficult to analyze that a static or dynamic test giving the capacity directly is necessary for a reliable design. *(As an aside, it is no more difficult to analyze a pile than a footing. Yet, even when a test has been performed establishing the capacity for a test pile, practice is usually limited to applying the so-found value to other piles at the site directly or by some depth or blow-count criterion. Unfortunately, a static back-analysis of the results is a very rare bird in current practice).*

Settlement analysis is usually only performed for footings in fine-grained soils. For footings in sand, the practice simply assumes that if the factor of safety is good enough, settlement is automatically taken care of and will not have to be further considered. As commonly stated, “we expect the settlement to be one inch or less”. For piles, settlement analysis is almost universally absent from conventional designs.

Bearing Capacity

The first bearing capacity formula was presented by Prandtl (1920) and it applied to material deforming plastically under stress, such as metal. A bearing capacity method of calculation for footings was proposed by Fellenius (1926) basing the analysis on cylindrical slip surfaces and shear resistance (τ) per the Coulomb relation ($\tau = c + \sigma \tan \phi$), employing the friction circle for analysis in frictional material. Terzaghi (1943) built on the Prandtl approach and presented a generally applicable formula, usually referred to as The Bearing Capacity Formula. Fig. 1 shows a page from Taylor (1948) referencing and comparing the Prandtl and Fellenius methods for clays. Fig. 2 quotes a figure from Bowles (1988) comparing the Terzaghi formula with adaptations by Meyerhof and Hansen.

The Fellenius Solution

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Because of their compressibility, soils do not show close agreement with Prandtl's hypothesis, which was originally set up for metals, and in actual cases of footings loaded to failure the region corresponding to Zone III is much narrower than that

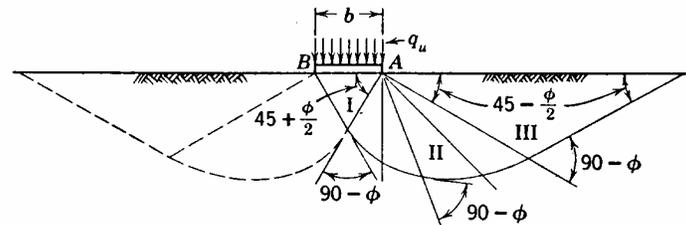


FIG. 19-6 Cross section illustrating Prandtl's plastic equilibrium theory.

shown in Fig. 19-6. However, the general concepts of the mechanics of failure given by this theory are reasonably correct.

19-10 The Fellenius Solution—Ultimate Bearing Capacity of Long Footings at or below the Surface of Highly Cohesive Soil

The Fellenius method of circular failure surfaces may be used to determine the ultimate bearing capacity of highly cohesive soils. The critical failure arc for a surface footing is shown in Fig. 19-7 (a) by a full line. It is seen that this surface agrees closely with the Prandtl failure surface for this case. It is not difficult to demonstrate that the circle shown is the critical one and that it furnishes for the ultimate bearing capacity of *long, surface footings on highly cohesive soils* the expression

$$q_u = 5.5c \quad (19-2)$$

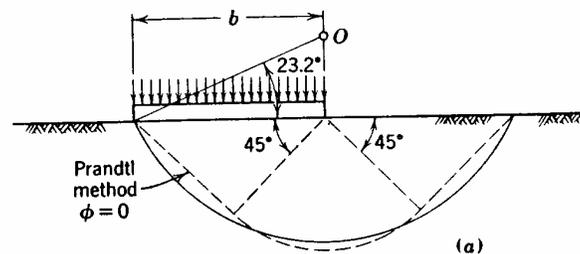


FIG. 19-7 Stability analysis of loaded footings on highly cohesive soil, based on the assumption of circular failure surfaces.

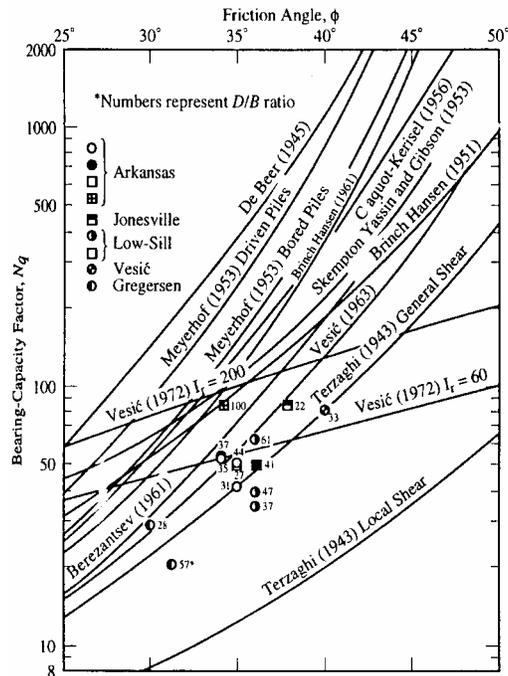


Fig. 3 Bearing Capacity Factors proposed by different authors (Coyle and Castello, 1981, as taken from Coduto, 1994)

Perhaps I need to prepare a first line defense by emphasizing that my statement pertains to the loading of footing or pile under long-term service condition. That is, fully drained conditions. Rapid loading of foundations in normally consolidated clays and silts will create pore pressures that reduce soil strength. Practice has established rules for the analysis of such “undrained” conditions, normally assessing the stability in a slip circle analysis with input of undrained shear strength. In less routine cases, the analyses consider actual stress-path behavior. Although the bearing capacity formula is still in use for these soils, its validity is very much in question.

No theoretical analysis or formula must be accepted as correct without first being compared to the response of full-scale tests. There is an abundant literature on results of tests on model footings. Full-scale tests on footings are rare, however. In contrast, the literature is full of reports on full-scale tests on piles. However, the analyses of these test have generally presumed that the pile shaft and the pile toe both respond to a loading according to the concept of bearing capacity due to shear failure. Well, the presumption is true for the pile shaft, but not the pile toe.

Results of Full-Scale Tests on Footings

Ismael (1985) performed static loading tests on square footings with sides of 0.25 m, 0.50 m, 0.75 m, and 1.00 m at a site where the soils consisted of fine sand 2.8 m above the groundwater table. The sand was compact as indicated by a N-index equal to 20 blows/0.3 m. The footings were placed at a depth of 1.0 m. The measured stress-settlement behavior of the footing is shown in the left diagram presented in Fig. 4. The right diagram shows the same data plotted as stress versus relative settlement, i. e., the measured settlement divided by the footing side. Notice that the curves are gently curving having no break or other indication of failure despite relative settlements as large as 10 % to 15 % of the footing side.

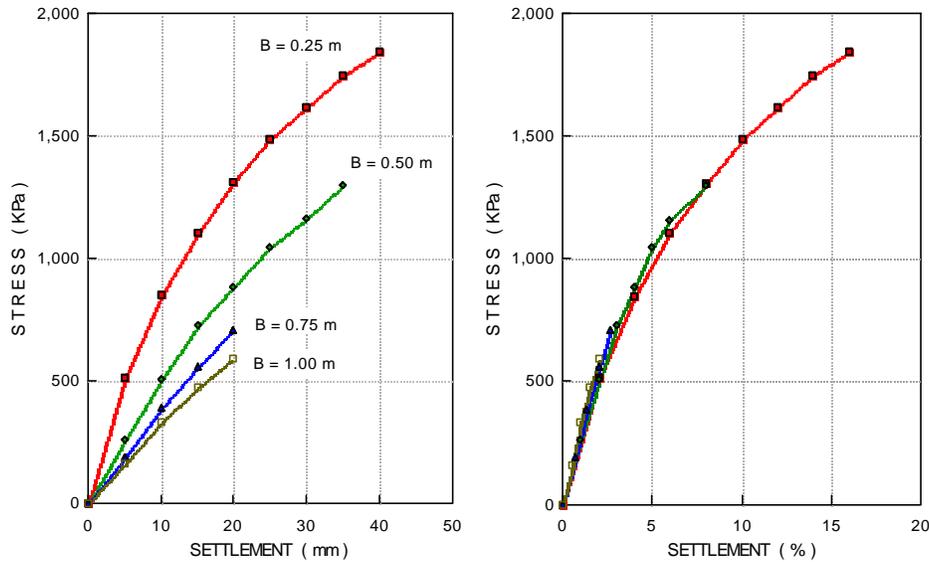


Fig. 4 Results of static loading tests on square footings in well graded sand
(Data from Ismael, 1985)

Similar static loading tests on square footings placed at a depth of 0.8 m in sand were performed by Briaud and Gibbens (1994) in a slightly preconsolidated, silty fine sand well above the groundwater table. The natural void ratio of the sand was 0.8. The footing sides were 1.0 m, 1.5 m, 2.0 m, and 3.0 m. Two footings were of the latter size. The results of the test are presented in Fig. 5, which, again, shows no indication of failure despite the large relative settlements.

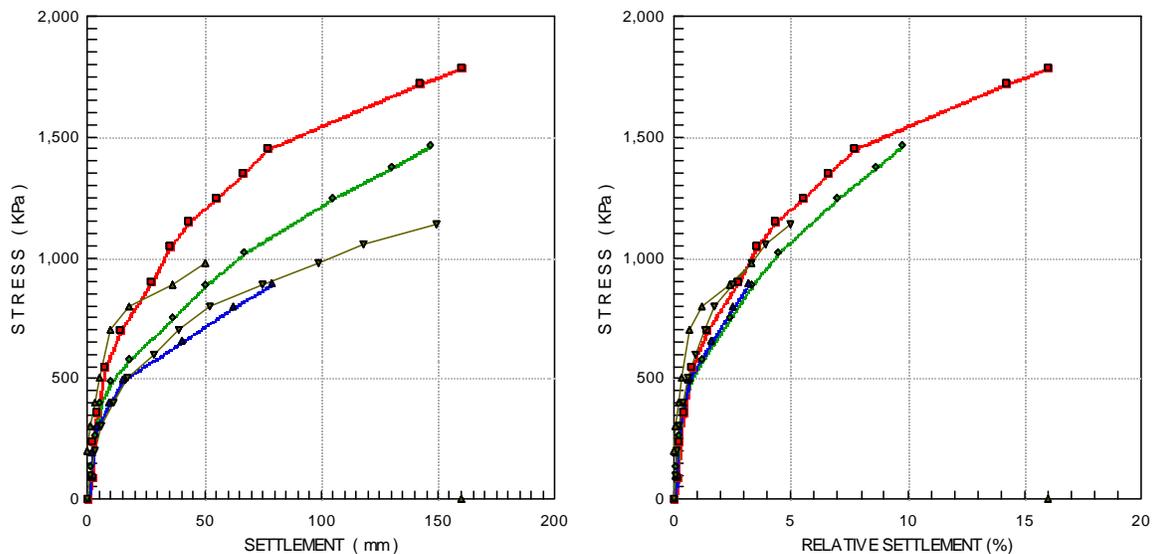


Fig. 5 Results of static loading tests on square footings in well graded sand
(Data from Briaud and Gibbens, 1994)

Results of Laboratory Tests on Footings

The results of the quoted full-scale field tests can be compared to laboratory test results on a 150 mm square footings performed by Vesic (1967). The tests were performed on the surface of a dry uniform

sand compacted to a void ratio ranging from 0.8 through 1.0. The results are presented in Fig. 6. The uppermost curve giving the results of the test on the dense sand differs in shape from the results of the tests on the loose sand, as well as from the tests on the full-scale tests (Figs. 4 and 5): instead of more or less rising gently, it reaches a peak and then drops off, suggesting a bearing capacity failure. However, the behavior of the test is not a failure in the sense of the bearing capacity equation. The test is performed in dense sand and, for the initial stages of loading, the sand dilates. That is, the volume of the sand increases in the beginning of the test (the footing still settles, because the sand can expand laterally). Later in the test, the dilatation ceases, the sand volume reduces, and the settlement increases.

The dilation takes place because the sand density is significantly smaller than that at the critical void ratio (as first discussed by Casagrande, 1935). The critical void ratio is a function of the mean soil stress. The smaller the mean stress, the larger the tendency for dilation. Consider the load-movement behavior of a footing placed on the surface of a soil body. The mean stress in the part of the soil body that is affected by the load from the footing is small. In contrast, the mean stress is much larger in the part of the soil body affected by a wide footing, because the affected zone lies deeper down where the overburden stresses are larger. Whether the footing is wide or small, if it is located deep down below the surface, the governing mean soil stress will be large. As in the case of a pile toe, for example.

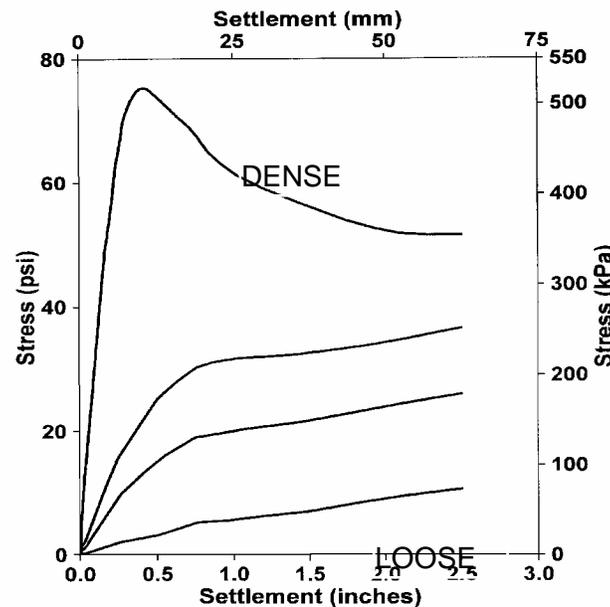


Fig. 6 Results of model tests on square footings in dry sand
(Data from Vesic, 1967, as taken from Vesic 1975)

Principles of Steady State Soil Mechanics

The principles explaining the behavior of the plate bearing tests shown in Fig. 6 are called Critical State Soil Mechanics or Steady State Soil Mechanics. These say, essentially, that it is not the density, or void ratio, alone that determines if a soil will dilate or not, it is the density or void ratio in relation to the steady state line (for details, see Altaee and Fellenius, 1994). The most characteristic parameters are the void ratio distance to the steady state line and the slope of this line in a void ratio vs. mean stress diagram. The distance has the symbol of the Greek letter upsilon, Υ , and is here called the “upsilon distance”. Others have called it the “e-prime” value or “the state parameter” (it is one of several

parameters, albeit an important one). The slope has the symbol of the Greek letter lambda, λ , and is always negative.

At a small mean soil stress, such as that for the case of a model footing on the surface of a soil body, even a loose soil would behave similarly to a very dense soil deeper in the soil mass. Fig. 7 (after Fellenius and Altaee, 1994) shows a void ratio versus mean stress diagram with a steady state line of a typical soil (actually, the line for a Fuji River sand, a Japanese standard sand). The diagram also shows one homologous point of void ratio and mean stress for a depth below each of three footings of diameters 0.5 m, 1.0 m, and 2.0 m placed at depths of one footing diameter below the ground surface. Three variations are considered:

- I. The three mentioned footings are placed in soils having equal void ratio at homologous points
- II. The 1.0-m diameter footing is placed in soils having equal mean stress, but different upsilon distances, at the homologous points
- III. The three mentioned footings are placed in soils of different void ratios chosen so that the upsilon distances is the same for the homologous points

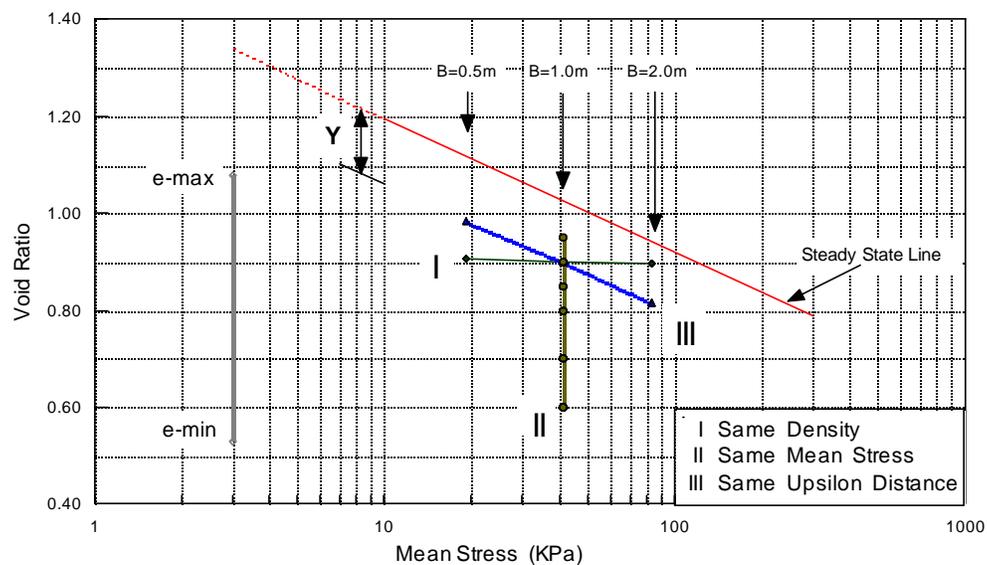


Fig. 7 Footing locations in the e - $\ln(p)$ diagram
 Analysis per steady state principles of tests on three square footings
 under different assumption of initial void ratio
 (Data from Fellenius and Altaee, 1994)

Fig. 8 shows the results of the computations for “tests” on the three footings when the soil has the void ratio (same density). The results are plotted as computed applied stress versus settlement for homologous points and versus relative settlement for the three footing sizes. The larger the size of the footing, the deeper the location of the homologous point and the larger the mean stress. Because of the sloping steady-state line, the wider footing will have a shorter upsilon distance. That is, it will behave “as were it in a looser soil” and it will show larger settlement for a certain applied stress. As in the full-scale test results, the relative settlements are very similar for the three footing sizes and no failure is indicated despite the up to 10 % relative settlement.

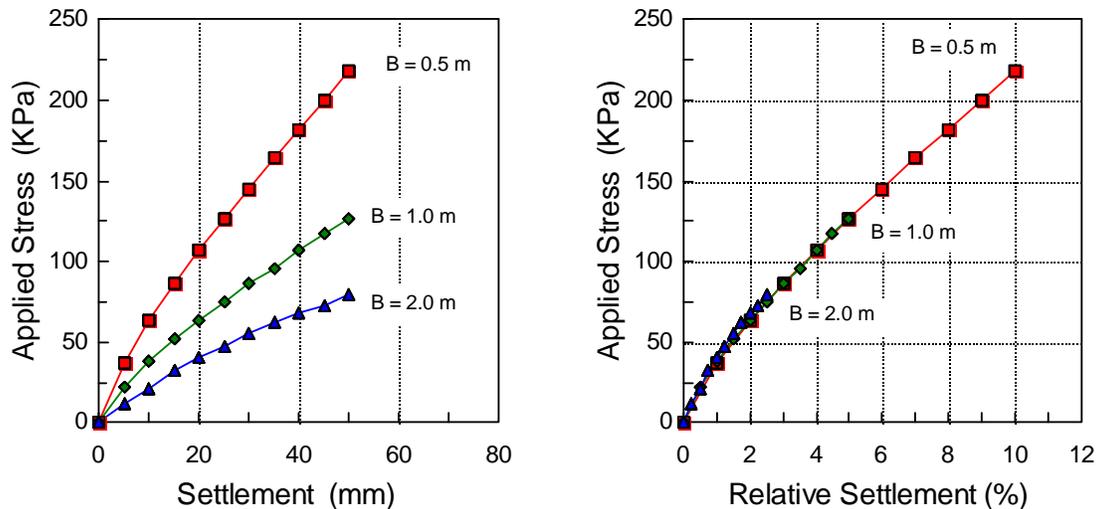


Fig. 8 CASE I

Stress vs. Settlement and Stress vs. Relative Settlement
(Data from Fellenius and Altaee, 1994)

Fig. 9 shows the results for a footing tested at successively larger epsilon distances. The curves show that, *for a given mean stress*, the larger the distance to the steady state line, or, if you prefer, the denser the soil, the stiffer the reaction. The important distinction is that density is defined in relation to the steady state line and not in terms of “relative density”, which is determined at very small mean stress (ground surface condition) and, therefore, of little use for conditions down in the soil where the mean stress is larger.

Fig. 10 shows that, *independent of the mean stress*, the stress-settlement behavior for the three footing sizes “tested” at equal epsilon distance is the same. As discussed by Altaee and Fellenius (1994), this analysis demonstrates the “law” of physical modeling, i. e., determining the behavior of a full-scale unit from a observations on a model-scale unit. Thus, the stress-settlement behavior of a model footing tested on the surface of a dense soil is representative for the behavior of a full-scale footing placed at some depth into the soil where the void ratio is such that the mean stresses for model and prototype are equal at homologous points. The behavior of, say, a 150-mm size plate tested on the surface of the sand is not directly representative for the behavior of the full-scale footing. For the model test to be representative for the full-scale footing, presumably much wider than the 150 mm plate, the model footing must be tested in a sand that is looser than the sand at the prototype footing. Frequently, tests on model footings are only representative for a full-scale footing in soils so dense that they are more like bedrock than soil. Model tests under ordinary gravity conditions are rarely representative for the real thing.

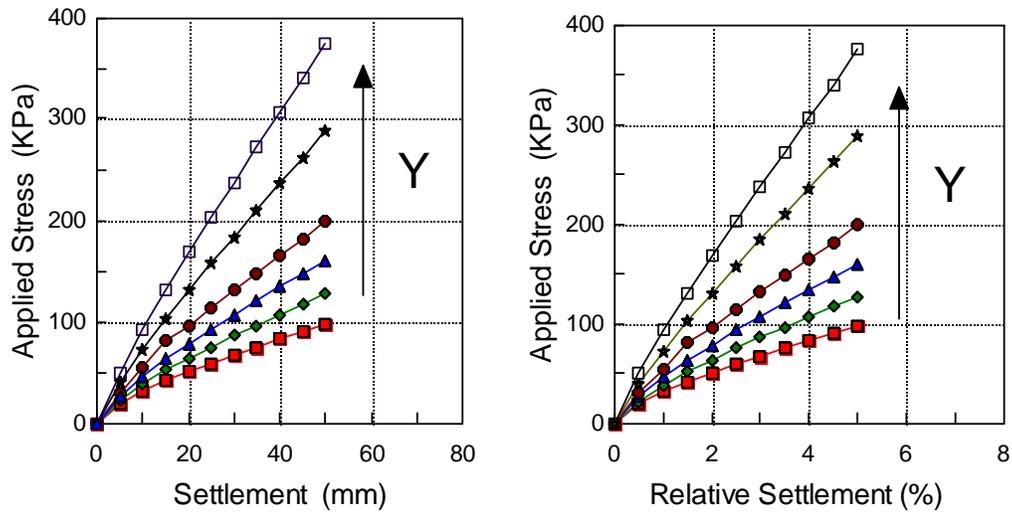


Fig. 9 CASE II
 Stress-Settlement for same mean stress at increasing Upsilon distance
 One metre wide square footings
 (Data from Fellenius and Altaee, 1994)

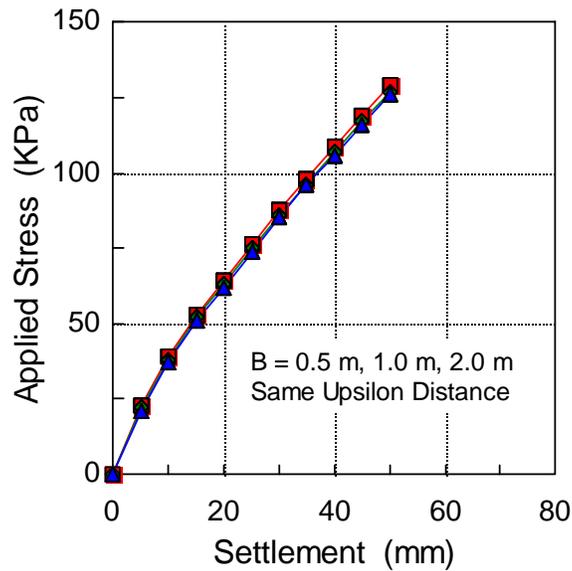


Fig. 10 CASE III
 Stress-Settlement for equal Upsilon distance
 (Data from Fellenius and Altaee, 1994)

Of course, some soils have a very flat steady state line, which means that the upsilon distance is pretty much the same for all mean stresses and extrapolation from a model to a prototype is direct. However, such soils are not very compressible and do not offer much problem for design.

Settlement Analysis

When settlement analysis is performed to supplement the bearing capacity analysis, it is usually performed by distributing the applied stress using 2:1 or Boussinesq methods and the soil compressibility is defined by either Young's modulus (sands) or by C_c - e_0 parameters. However, if the analysis is made assuming normally consolidated soils, the calculated stress-settlement curve will bend upward, directly contrary to observed behavior (Figs. 4 and 5, for example). With an investment of some effort, a match between observed and calculated curves can be made if the analysis assumes that the soil is overconsolidated and that the degree of overconsolidation reduces within the affected depth. There is a correlation between overconsolidation and the mean stress difference between the void ratio/mean stress point and the horizontal distance to the steady state line. However, the correlation is only approximate. The conventional settlement analysis assumes one-dimensional behavior and neglects the contribution to the settlement from lateral movement of the soil under the footing, which movement extends to the outside of the footprint area.

The load-movement or stress-settlement behavior of a footing or a pile toe, such as shown in Fig. 4, can be described by Eq. 2.

$$\sigma_1 = \sigma_{\text{ref}}(\delta_1/\delta_{\text{ref}})^e \quad (2)$$

where:

- σ_1 = applied stress
- σ_{ref} = reference stress
- δ_1 = observed settlement (or relative settlement)
- δ_{ref} = observed settlement (or relative settlement) for the reference stress
- e = an exponent

It is simple to apply the relation to the case shown in Fig. 4. Taking the stress at a relative settlement of 10 % as reference, the exponent, e , becomes about 0.6. To do the same for the case shown in Fig. 5, the data must first be adjusted for the preconsolidation, which requires solving for a constant added to the settlement input. Typically, the exponent, e , ranges between 0.5 through 0.9. Using the Eq. 2 relation in a curve-fitting approach can often be a more reliable tool for applying observed load-settlement data to a design as opposed to a back calculation of compressibility parameters or other theoretical analyses.

Results of Full-Scale Tests Deep into the Soil

The foregoing has presented the behavior of footings tested at shallow depths. It may seem that results from tests on small diameter footings at larger depths would be rare. However, every pile test is a test on a small diameter footing located at a depth many times larger than the diameter. Unfortunately, the load applied to the pile toe is usually not known very accurately and neither is the movement. Even when a load cell or similar is placed at the pile toe and a telltale is used to measure the pile toe movement, the test data are difficult to interpret, because the zero load (the starting load) is not known. A bit over ten years ago, a testing method was developed that eliminated these difficulties: The Osterberg Cell (Osterberg, 1998), which is a great new tool for use by the geotechnical engineer. The method consists in principle of placing a hydraulic jack at the pile toe and expanding the jack while measuring the imposed load and the resulting toe movement. The result of the test, amongst other, is the load-movement curve of the pile toe.

Fig. 11 presents the results of an O-cell test on a 2.8 m by 0.8 m, 24 m deep barrette in Manila, Philippines. The soils at the site are silty sandy volcanic tuff. The diagram shows both the downward movement of the barrette base (toe) and the upward movement of the barrette shaft. The load indicated for the toe includes the buoyant weight of the barrette, whereas this load has been subtracted from the shaft loads before plotting the data. While the barrette shaft obviously has reached a ultimate resistance state, no such state or failure mode is evident for the barrette toe.

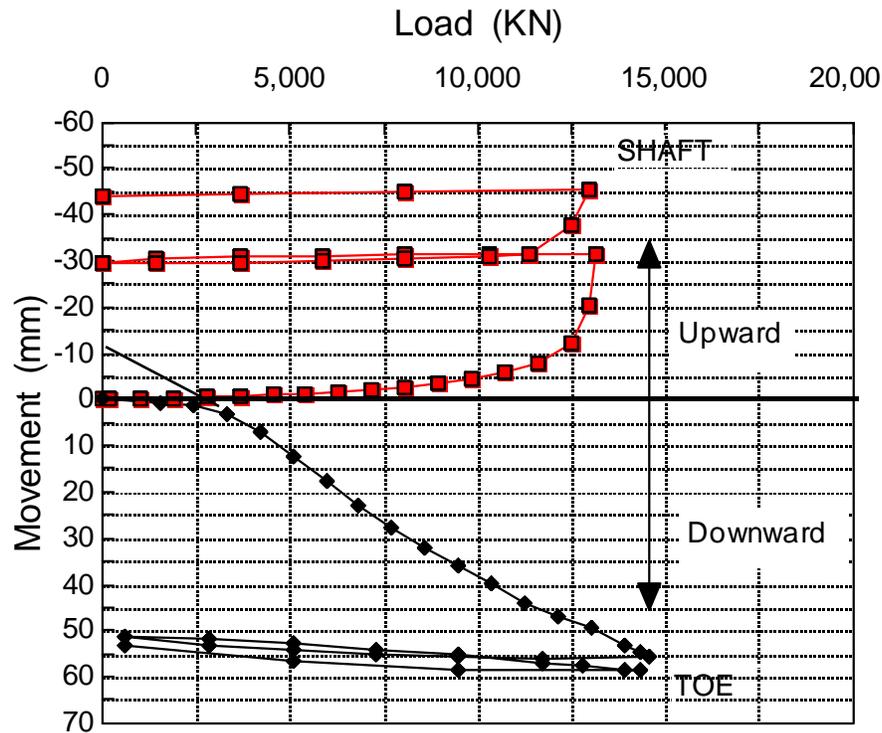


Fig. 11

Result of an O-Cell Test on a 2.8 m x 0.8 m, 24 m Barrette at Belle Bay Plaza, Manila
(Test Data from LOADTEST Inc. Report dated August 1997)

At the start of the test, the barrette is influenced by residual load. The solid line added to the beginning of the toe movement indicates an approximate extension of the toe movement to the zero conditions. The residual load is about 2,500 KN and the soil at the toe are precompressed by a toe movement of about 10 mm. The maximum toe movement is 65 mm. The relative movement is 2 % or 8 % depending on which cross section length one prefers to use as reference to the barrette diameter.

Fig. 12 presents the results of an O-cell test on a 2.5 m circular shape pile, 85.5 m into a clayey silty soil in Vietnam. The steep rise over the first 25 mm movement as opposed to the behavior thereafter appears to suggest that a failure value could be determined. Say, by the intersection of two lines representing the initial and final trend of the curves. However, this would be an error. Also this pile (as all piles, really) is prestressed by the surrounding soils to a significant residual load. As shown in the diagram, an approximate extension of the curve intersects the abscissa at a movement of about -50 mm. The apparent residual load is about 10 MN and amounts to about one third of the ultimate shaft resistance (determined in the O-cell test). Compare also the two unloading and reloading cycles. The reloading curve shows that the residual load increased for each such cycle.

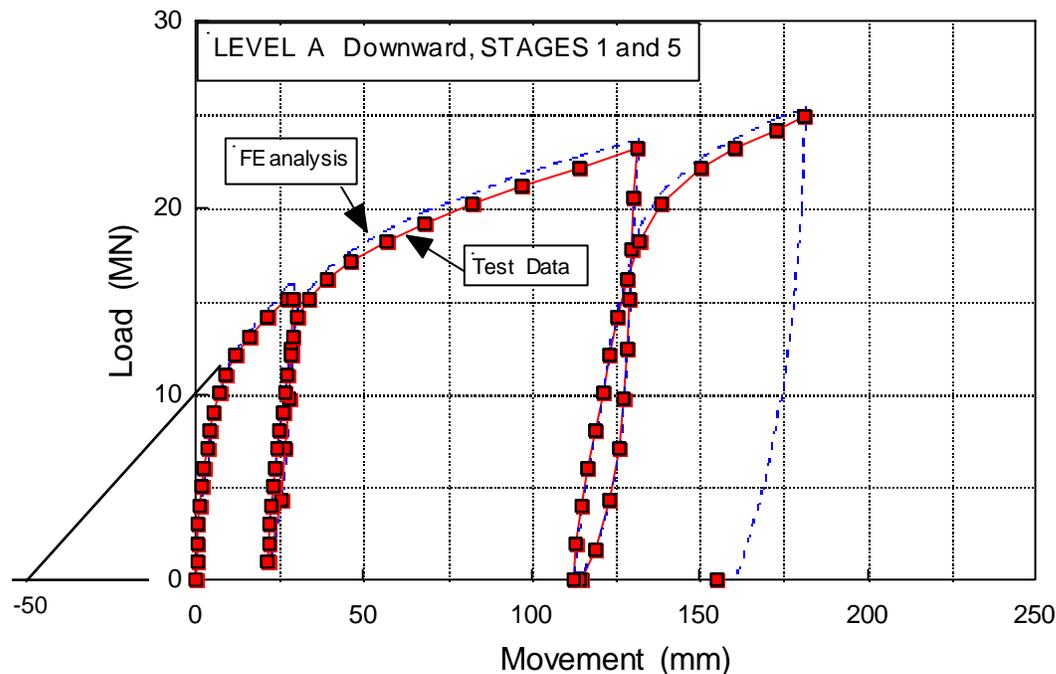


Fig. 12

Result on an O-Cell Test on a 2.5 m diameter, 85.5 m Long Pile

at My Thuan Bridge, Vietnam

(Test Data and Finite Element Analysis Results
from Urkkada Technology Report, May 1998)

The maximum toe movement is 180 mm. The relative movement is 7 % (or 9 % if the precompression movement is included). No failure is evident from the test records.

Fig. 13 presents the results of an O-cell test on another 2.8 m by 0.8 m, barrette in Manila, Philippines. The barrette is slightly deeper, 28 m. The results are very similar to those for the first barrette, including the estimate of the residual load.

Fig 14 presents the results of a static loading test performed on a 20 m long pile instrumented with a telltale to the pile toe for measuring the pile toe movement. The pile toe load was not measured. The load-movement diagram for the pile head shows that the pile clearly has reached the ultimate load. In fact, the pile “plunged”. Judging by the curve showing the applied load versus the pile toe movement, it would appear that also the pile toe reached an ultimate resistance — in other words, attained its bearing capacity. However, this is not the case as will be discussed using the plot shown in Fig 15.

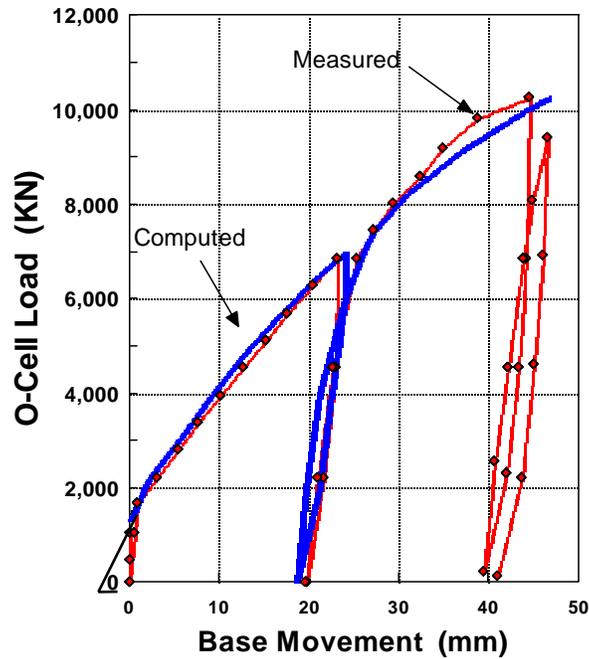


Fig. 13

Result of an O-Cell Test on a 2.8 m x 0.8 m, 28 m Barrette at Alfaro's Peak, Manila (Fellenius et al., 1999)

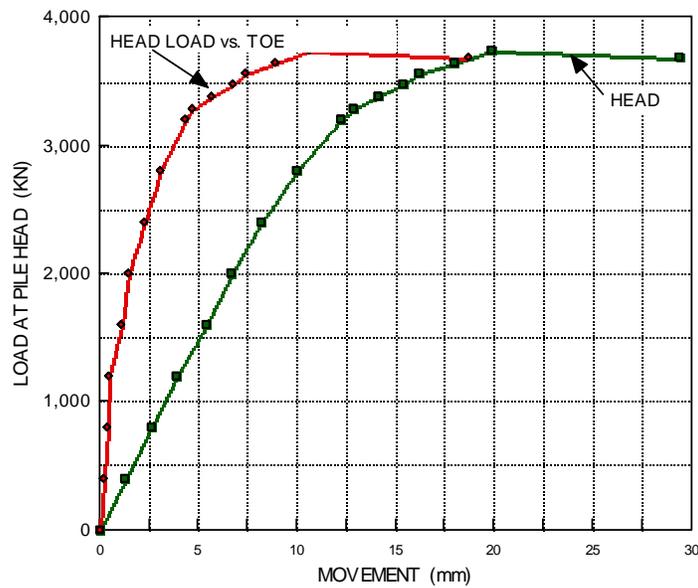


Fig. 14

Load-movement diagram of a static loading test on a 20 m long, 450 mm diameter closed toe pipe pile in compact sand with telltale measurements of toe movement

(Data from F. M. Clemente, TAMS Consultants)

Most of the shaft resistance was probably mobilized at a toe movement of about 2 mm to 3 mm, that is, at an applied load of about 2,500 KN. At an applied load beyond about 3,300 KN, where the movements start to increase progressively, the shaft resistance probably started to deteriorate (strain softening). Thus, approximately between pile toe movements of about 3 mm to about 10 mm, the shaft resistance can be assumed to be fully mobilized and approximately constant. An adjacent test indicates that the ultimate shaft resistance was about 2,000 KN. When subtracting the 2,000 KN from the total load over this range of toe movement, the toe load can be estimated. This is shown in Fig. 15, which also shows an extrapolation of the toe load-movement curve beyond the 100-mm movement, which implies a strain softening behavior of the pile shaft resistance. Extrapolating the toe curve toward the ordinate indicates the existence of a residual load in the pile prior to the start of the static loading test.

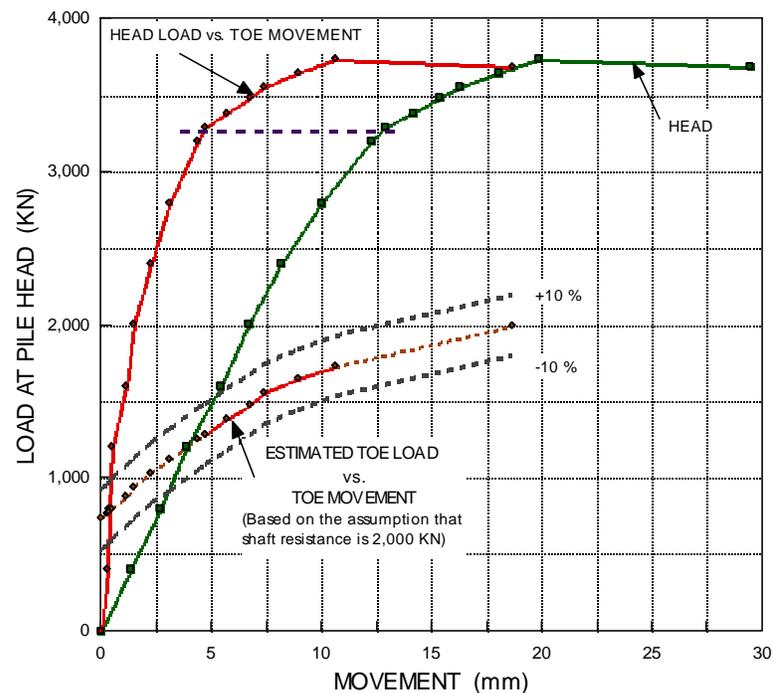


Fig. 15

Fig. 14 with the results of analysis of the load-movement of the pile toe

[Reference is made of the results from a static loading test on an adjacent pile instrumented with strain gages indicating a shaft resistance of 2,000 KN]

A back calculation of the toe movement curve and a simulation of the observed pile load-movement behavior can quite easily be performed by means of so-called t-z curves to represent the soil response along the pile shaft and at the pile toe (for the pile toe, the term should really be “q-y curve”). Eq. 2 is a useful such t-z curve. In this case, the shaft curve would reference the shaft resistance at a movement of about 3 mm and with an exponent of about 0.15. The toe curve would reference a toe movement of 10 mm and apply an exponent of 0.5. The latter indicates a toe load-movement behavior that does not show an ultimate value.

Discussion and Conclusions

Full-scale tests on footings and piles supported by theoretical analysis based on soil response to a stress increase show that bearing capacity failure does not occur for footings and pile toes. Even for settlements much in excess of 10 % of the footing diameter, the soil response is not that of the footing approaching or reaching an ultimate failure mode, but of soil stiffness. For this reason, the conventional bearing capacity formula does not correctly represent foundation behavior. Moreover, to apply an ultimate resistance approach (whether in a global factor of safety or limit state conventions) is fundamentally wrong. Instead, the approach should be to calculate the settlement of the footing considering the serviceability of the foundation unit.

The pile toe load-movement behavior is a function of the compressibility of the soil at the pile toe. When the behavior is actually determined in a test, the picture is not as clear as in the case of a footing. The pile toe behavior is influenced by that set-up after installation loads the pile shaft and transfers load to the pile toe. In case of driven piles, the driving may have provided considerable precompression to the soil at the pile toe. A load-movement curve, if established, might be interpreted to show a “failure” point when in reality the point only reflects the precompression value. The stress-settlement or stress-movement behavior of a pile toe should be analyzed as the compressibility problem it really is, not by bearing capacity formulae or similar reasoning.

Ordinarily, a conventional static loading test on a pile cannot disclose the amount of residual load present in the pile before the start of the test. Indeed, the tests results pertain to the pile head load-movement and one cannot easily separate the shaft resistance from the toe resistance without resorting to special instrumentation. In many tests, the shaft resistance breaks down progressively as the applied load increases. The breakdown increases the movement of the pile head, which then is interpreted as a progressively increasing movement of the pile toe for each load increment. Tests on instrumented piles where toe load and toe movement have been determined show results that are similar to those obtained by the O-cell (when the residual load is taken into account).

Dynamic tests on piles are taken during the impact driving of a pile. Usually, the maximum pile toe movement for the blow is about 5 mm to 15 mm. The pile toe resistance is then approximately the load required to move the pile that same distance. This resistance is not equal to a pile toe bearing capacity. Were it the ultimate toe resistance, moving the pile toe additionally would not require much additional load. The actual situation is the opposite. However, the point is of little practical importance as the short duration impact only provides that limited movement. For an O-cell test, the concept of bearing capacity just does not apply and is not needed. For a Statnamic test, it is very important to ensure that the pile toe has not been moved too much in the test, or the capacity might be overestimated. However, the implications for the testing of piles lie outside the scope of these notes and will not be addressed further.

Geotechnical investigation should include information on the compressibility characteristics of also coarse-grained soils and geotechnical design should place the greater emphasis on determining the settlement of the foundations.

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