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## WAVE EQUATION ANALYSIS AND DYNAMIC MONITORING OF PILE DRIVING

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**Abstract** The wave equation analysis of driven piles is presented with a comparison of the Smith and Case damping approach and a discussion of conventional soil input parameters. The cushion model is explained, and the difference in definition between the commercially available computer programs is pointed out. Some views are given on the variability of the wave equation analysis when used in practice, and it is recommended that results should always be presented in a range of values as corresponding to the relevant ranges of the input data.

A brief background is given to the Case-Goble system of field measurements and analysis of pile driving. Limitations are given to the field evaluation of the mobilized capacity. The CAPWAP laboratory computer analysis of dynamic measurements is explained, and the advantages of this method over conventional wave equation analysis are discussed. The influence of residual loads on the CAPWAP determined bearing capacity is indicated.

This paper gives a background to the use in North America of the Wave Equation Analysis and Dynamic Monitoring in modern engineering design and installation of driven piles. The purpose of the paper is not to provide a comprehensive state-of-the-art, but to present a review and discussion of aspects, which practising civil engineers need to know in order to understand the possibilities, as well as the limitations, of the dynamic methods in pile foundation design and quality control and inspection.

### The Wave Equation Analysis

Longitudinal wave transmissions and solutions to wave equations have been known by mathematicians for almost a century. Fifty years ago, Isaacs (1931) pointed out that wave action occurred in a pile after impact and that wave mechanics could be used to analyse pile driving. A solution to the one-dimensional wave equation, as applied to pile driving, was first published by Glanville et al. (1938). However, before the existence of computers, the solutions were not practical. About thirty years ago, E. A. L. Smith

developed a mathematical model for an analysis suitable for computer solution. Later, he also developed a computer program, which has become the basis for all modern wave equation computer programs (Smith, 1960).

The one-dimensional wave equation is derived by applying Newton's Second Law to a short pile element. The wave equation is a second degree differential equation, which solution is obtained by integration. Because of the complex boundary conditions, however, a direct solution is difficult and impractical. Therefore, to enable a rational solution with clearly defined boundary conditions, Smith (1960) separated masses and forces in a mathematical model of the pile, the hammer, the capblock, the cushion, and the soil, as shown in Fig. 1. The model consists of a series of mass elements connected with weightless springs and subjected to outside soil forces. The mass elements are infinitely stiff. Their actual stiffness is represented by the weightless spring, which has a stiffness equal to  $EA/L$ , where  $E$  is the modulus of elasticity of the material,  $A$  is the cross sectional area, and  $L$  is the length of the mass element.

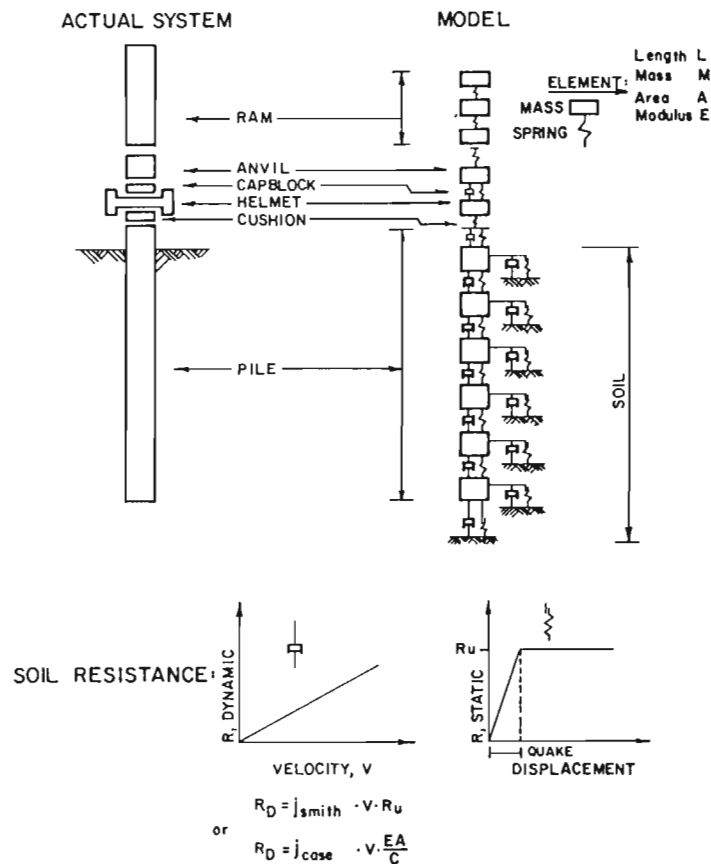


FIG. 1

Actual piling system and Smith mathematical model of hammer, capblock, cushion, pile, and soil (Goble and Rausche, 1976).

According to Smith (1960), in the soil along each pile element and at the pile tip, the soil forces are modelled to consist of a static resistance, which is an elasto-plastic function of displacement, and a dashpot dynamic damping resistance, which is a linear function of the pile velocity.

Initially, the static soil resistance force increases linearly with the displacement of the pile. At a certain displacement called the quake, the force reaches a maximum value. Thereafter, the soil resistance is plastic, that is, continued displacement requires no additional static force.

The quake is usually assumed to be 2.5 mm, but can vary within a very wide range (Authier and Fellenius, 1980b). Naturally, the quake along the pile shaft and at the pile end can differ. Furthermore, the soil stiffness (the slope of the elastic portion) may or may not be equal in loading and unloading.

In Fig. 2, the assumed ideal behaviour of the static soil resistance is compared with the real static load-deformation curve (after Goble and Rausche, 1976). The amount of energy dissipation is represented by the shaded areas. As the area of dissipated energy of the model should equal that of the reality, the quake in the model is set smaller than the real quake.

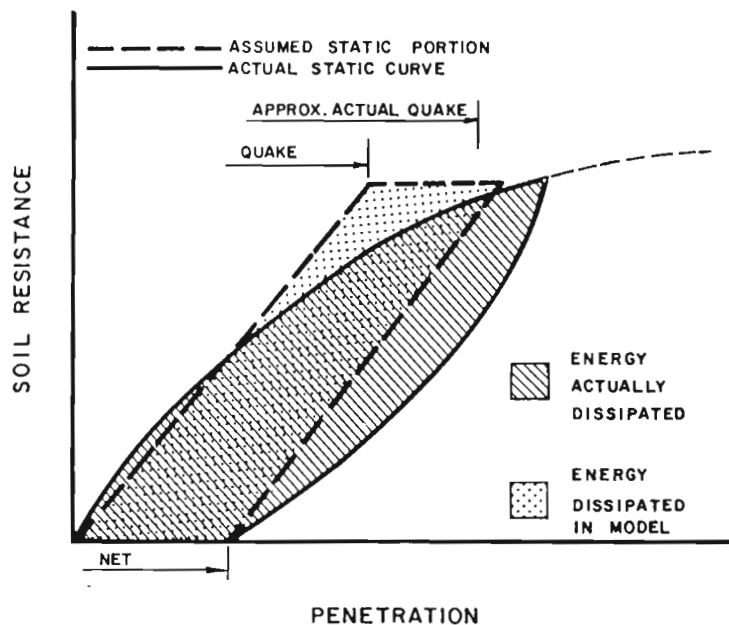


FIG. 2

Actual and model of static soil resistance (Goble and Rausche, 1976).

The linear proportionality of the damping resistance with the pile velocity, as assumed by Smith (1960), has been shown to be incorrect in clay soils. See, for instance, Litkouhi and Poskitt (1980). However, the assumption of linearity is considered an acceptable simplification in most practical cases.

Fig. 3 shows typical static and dynamic portions of soil resistance - both separately and combined to a total soil resistance. A corresponding typical "set-rebound graph", i.e., graph of penetration with time, is also shown in the figure.

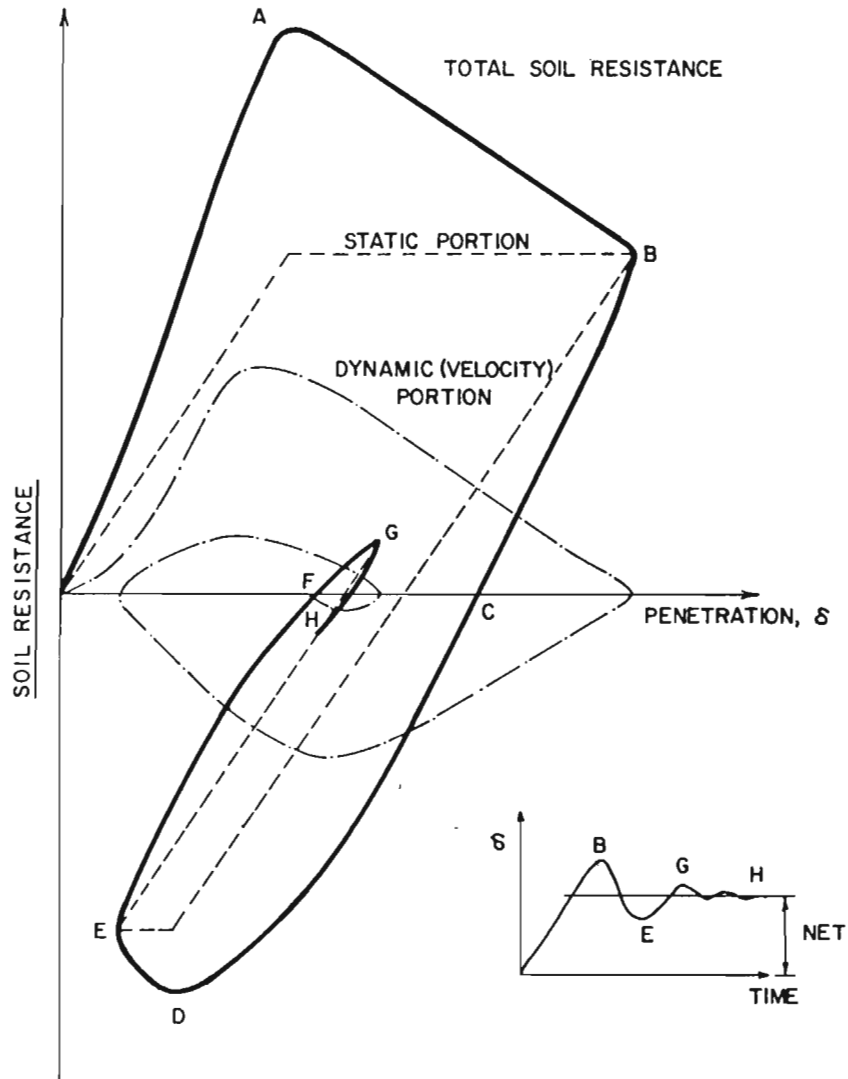


FIG. 3

Modelling of soil resistance - static, dynamic, and total - using Case damping.

The ratio between the velocity and the damping force is called the damping factor. It is usually denoted "j". Toe, or tip, damping,  $j_t$ , acts at the pile tip. Skin, or shaft, damping,  $j_s$ , at the pile shaft. In the original Smith model, the dimension of the damping factor is inverse velocity, time/length, and the damping force generated is equal to the damping factor times the velocity of the pile element times the activated static soil resistance.

The Smith damping force will increase, as the soil resistance increases. However, when the soil resistance increases, velocity is usually reduced, and a high velocity in a low strength soil can give a damping force similar to what is obtained from a smaller velocity in a stronger soil. Consequently, the Smith damping factor is rather insensitive to the soil type, or to the soil dynamic properties. (For a parametric study of the influence of the damping factor see Ramey and Hudgins, 1977).

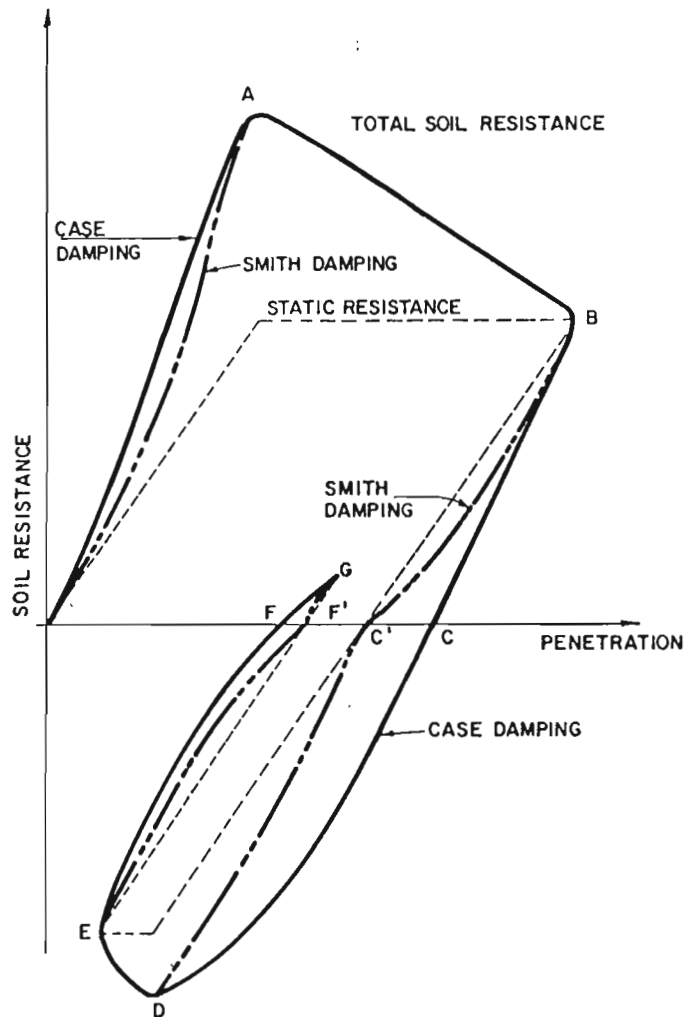


FIG. 4

Comparison of total soil resistance determined with Case and with Smith damping.

In an approach by Goble and Rausche (1976), the damping factors are dimensionless and the damping force is obtained by multiplying the velocity with the damping factor and the pile-material impedance,  $EA/c$ , where  $c$  is the wave velocity in the pile. This damping factor is called "Case damping factor". In contrast to Smith damping, the viscous Case damping can more easily be related to the soil properties. However, with Case damping, the soil damping force becomes dependent also on the particular pile material and cross sectional area.

The dynamic portion of the soil resistance shown in Fig. 3 has been determined using Case damping. Had Smith damping been used, instead, the results would have been slightly different. Fig. 4 shows a comparison between the total resistance using Case and Smith damping as applied to the example in Fig. 3. The assumption has been made that the two resistances are equal at point A in the diagram. As shown, Smith damping results in the dynamic resistance being zero when the static resistance is zero - point C' in the diagram - although the pile has a velocity at this point.

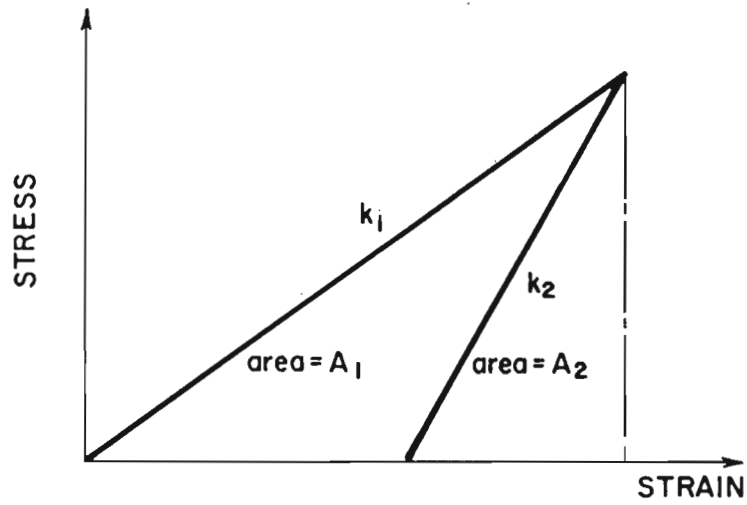
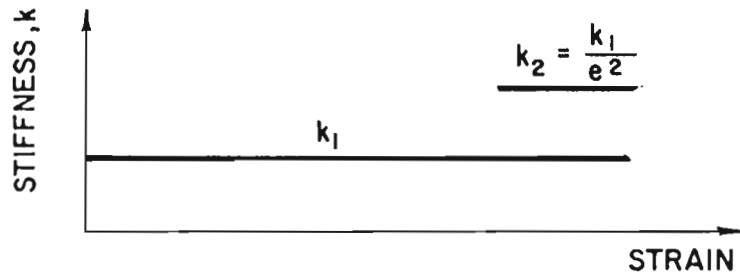
For most engineering materials, the elastic modulus varies with the stress level and depends on whether the load causing the elastic deformation is increasing or decreasing. That is, the elastic modulus is in practice never truly constant, nor is the average slope in loading truly parallel to the one in unloading. This difference is usually considered insignificant in practice for common pile materials, such as steel and concrete. However, for wood and other materials used in capblocks and cushions, and when going from one unit to another in the driving system (e. g. from hammer to anvil), the difference in stiffness in loading and unloading causes a loss of energy that is far from insignificant.

The stress-strain behaviour of cushion materials is illustrated in Fig. 5 showing an idealized linear behaviour, i.e., constant stiffness, but with a difference in loading and unloading stiffness causing an energy loss. The relative energy loss is given as a coefficient of restitution,  $e$ , defined as the square root of the ratio between the energy leaving the material ( $A_2$ ) and the energy given to the material ( $A_1 + A_2$ ). It can also be related to the stiffness (slope) in loading ( $k_1$ ) and in unloading ( $k_2$ ). Then, the coefficient of restitution is equal to the square root of  $k_1/k_2$ .

The value of the coefficient of restitution is usually taken as 0.85 in a Wave Equation Analysis, but it can vary appreciably from this value. A value of 0.85 means that the stiffness in loading is 72 % of the stiffness in unloading, or that area  $A_1$  is 28 % of area  $A_1 + A_2$ , or that almost 30 % of the energy is lost in that particular unit. It is, therefore, not surprising that actual measurements show that sometimes more than 50 % of the nominal energy of a hammer is lost in the anvil-capblock-helmet-cushion-system.

The energy lost is dissipated in the form of heat. For instance, after prolonged use, a wood cushion burns and must be replaced. However, before burning, the cushion has become so compressed, dried out from the heat, and hardened that its stiffness can have increased by an order of magnitude and more.

In most computer Wave Equation Programs,  $k_1$  and  $e$  are input, and the computer calculates  $k_2$ . However, in one commercially available program, the WEAP program, which is discussed later,  $k_2$  is input, and  $k_1$  is calculated from  $k_2$  and  $e$ . This program uses, in addition, the more elaborate model of the stress-strain behaviour of materials shown in Fig. 6.



ENERGY INPUT =  $A_1 + A_2$

ENERGY OUTPUT =  $A_2$

COEFFICIENT OF RESTITUTION  $e = \sqrt{\frac{A_2}{A_1 + A_2}} = \sqrt{\frac{k_1}{k_2}}$

FIG. 5

Stress-strain diagram defining stiffness,  $k$ , and coefficient of restitution,  $e$ .

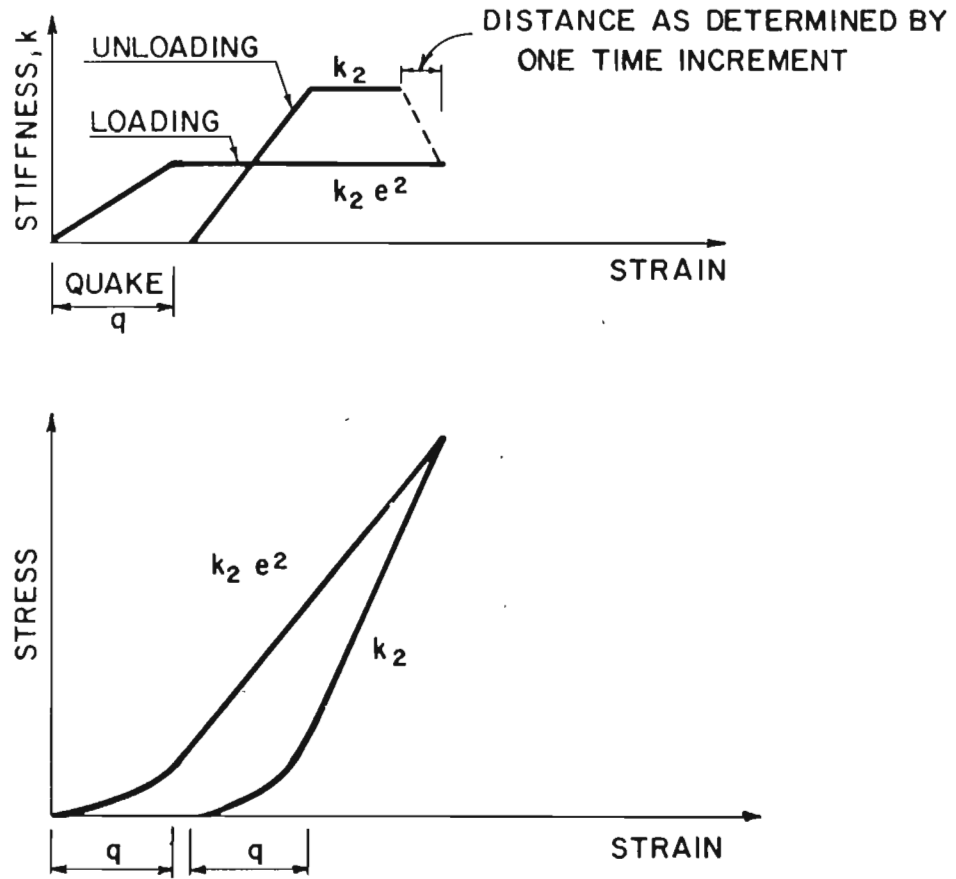


FIG. 6

Stress-strain diagram as used in the WEAP program.



The Smith model permits all parameters in the analysis to be treated separately. Hence, the effect can be isolated of a change of one parameter of one element on the neighbouring elements, as well as on the entire calculation.

The calculation begins by giving the hammer ram an initial velocity. A pile head displacement during a specific short time interval is calculated from the integral of the velocity over this time interval. The displacement compresses the uppermost spring, and the resulting force is calculated using the particular spring constant, which is the given element stiffness. The acceleration of the next element is then computed using the resultant force and the element mass, which then gives the element velocity at the end of the time interval. Element by element, and time interval (increment) after time interval, the computation is carried down the pile. Using the computed displacements and velocities, the spring forces acting on each mass element are determined from the spring deformations - pile and soil - and from the dashpot damping forces.

For a given application, a series of ultimate static soil resistance forces,  $R_u$ , damping factors,  $j$ , and quake values,  $q$ , are assigned at each element. Then, the ram is given its rated impact velocity, whereupon the computer takes over continuing the dynamic computations through successive time increments until all element forces are smaller than the  $R_u$ -value assigned to the particular element. The resulting total permanent displacement, and the sum of all individual element  $R_u$ -values give a point on a  $R_u$  versus displacement curve in a diagram called the "Bearing Graph". In this procedure, the permanent displacement (or "Blow-count") is determined, as resulting from a series of assigned total resistances defining the shape of the Bearing Graph. However, the diagram is plotted, by tradition, with the blow-count as the independent variable.

In addition to the Bearing Graph, the Wave Equation Analysis gives stresses in the pile and driving energy developed in the pile, which also can be represented as a function of the blow-count. Naturally, stresses, forces, and movements at different depths in the pile, and at different times after impact, etc., can equally well be obtained as output from the computer.

There are a few commercially available computer programs for the Wave Equation Analysis of pile driving. The programs most widely known and used in North America are the TTI program (Hirsch et al., 1976) and the WEAP program (Goble and Rausche, 1976). (An earlier version of the TTI program was presented by Lowery et al., 1967).

The TTI program originates in the approach by Smith (1960), but is modified to accommodate a large variation of field problems. It is developed primarily for analysis of piles driven with air/steam hammers or drop hammers. Diesel hammers are simply modelled as drop hammers with an explosive force acting in conjunction with the impact.

The TTI program uses Smith damping. However, as shown in Fig. 4, the strict Smith damping approach results in a zero damping force in unloading, when the static resistance is zero. This is undesirable, because at this point, the pile still has a velocity. The program has corrected for this in a special calculation that brings the total soil resistance closer to the results obtained by viscous damping and similar to the B-C-D shape shown in Fig. 4.

The WEAP program was developed in response to some shortcomings of the TTI program with regard to piles driven by a diesel hammer. The WEAP program models the actual combustion sequence of the diesel hammer considering the volume of the combustion chamber and the fuel injection. The program also calculates the ram rebound of the hammer. When the rebound distance does not agree with the original downward travel of the ram, the analysis is repeated with a new initial ram travel until agreement is achieved. Another development is the more representative stress-strain model

mentioned above and in Fig. 6. Furthermore, the WEAP program allows the alternative use of both Smith damping and viscous Case damping.

The WEAP program has most of the currently available diesel hammers on file, which makes it simple to run an analysis. Also, current air/steam hammers are on file, and the program is equally well suitable for these hammer types, as well as for drop hammers.

The reliability of the Wave Equation Analysis depends on the reliability of the dynamic and static soil parameters assumed as input values, i.e., coefficients of restitution, damping factors, static resistance distributions, and quake values. See, for instance, the parametric study by Ramey and Hudgins (1977). However, accurate input values necessitate knowledge of representative dynamic properties of the entire system, i.e., the hammer and its efficiency, the capblock, the cushion, the pile, and its components, as well as of the soil. Then, the analysis is still susceptible to common occurrences, such as improperly performing hammers, use of inadequate cushions and capblocks, eccentricities in the leads arrangements, etc.

In other words, the analysis necessitates an experienced operator with thorough knowledge of not only computer work and piling practice, but also of soil mechanics and actual behaviour of soils in practice.

The reports by Hirsch et al. (1976) and Goble and Rausche (1976) contain recommendations for input values to use for specific conditions. With the recommended input values, the analysis results in at least a qualitatively correct picture of the driving, which is far more in agreement with reality than a calculation produced by means of ordinary pile driving formulae. The Wave Equation Analysis will indicate for a given pile the most suitable driving criteria and be immensely valuable, when comparing different pile hammers.

However, a single Wave Equation Analysis run will only by accident give a quantitatively correct prediction of the pile driving results. To account for variability in the field, when performing a Wave Equation Analysis for use in an actual case, it is necessary that several computer runs be made using a range of applicable input parameters to result in a Bearing Graph, for instance, in the shape of more or less wide bands rather than a single curve.

Fig. 7 presents an example of an attempt to account for the variability in the field. The results are given in two bands. One represents a normal hammer efficiency, and one represents a suspected lesser efficiency. The upper and lower bounds of each band have been determined using two realistic ranges of quake values and damping factors.

The practical capacity of the applied hammer-pile-soil system is indicated by the start of the flattening out of the curve. As shown in Fig. 7, this occurs within a range of ultimate static resistance of 650 KN for the lower bound of the lower band and 1100 KN for the upper bound of the upper band. Without having actual measurements and/or previous experience from the actual hammer used for the actual pile at a similar site, if not the actual site, such a variation must be expected, when trying to analyse a practical case.

#### The Dynamic Monitoring and the Case-Goble system

The current knowledge of the dynamic properties of the hammer-capblock-cushion-pile-soil system selected for input in a Wave Equation Analysis is not adequate to enable an accurate quantitative prediction of, for instance, the Bearing Graph. However, most of the difficulties can be eliminated by measuring and studying the strain (stress) waves generated in the pile. The most well known and widely accepted system of measurement

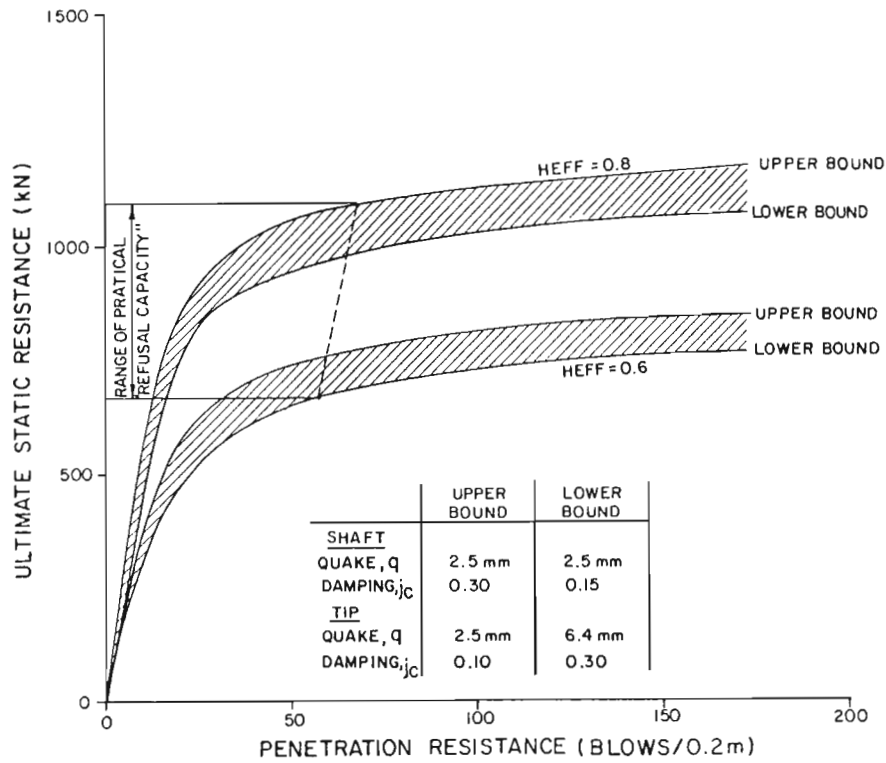


FIG. 7

Results of Wave Equation Analyses of a closed-end pipe pile driven with a drop hammer. Comparison is made between two hammer efficiencies (HEFF), and a range of quake values and damping factors. ( $W = 28 \text{ KN}$ ,  $H = 1.0 \text{ m}$ ,  $L = 12.2 \text{ m}$ ,  $D = 275 \text{ mm}$ ,  $RULT\text{-}SHFT = 450 \text{ KN}$ ).

is the Case-Goble system developed by Case Western Reserve University, and Goble and Associates, Cleveland (Goble et al., 1970, and Goble et al., 1980).

The Case-Goble system makes use of independent measurements of strain and acceleration taken in the field during actual pile driving. (The monitoring equipment is described below). The strain is directly converted to force, and the acceleration is integrated to obtain the velocity of the pile. The measurements are taken by means of a "Pile Driving Analyser System", which consists of several separate units. The actual Analyser is a preprogrammed field computer developed by Pile Dynamics Inc., Cleveland. When monitoring the driving of a pile, the Analyser is kept in a monitoring station on the ground, which is connected via a cable and a connector box to the measuring gauges attached to the pile. The Analyser is also connected to two auxillary instruments: a storage oscilloscope and a minimum four channel analog tape recorder.

The gauges consist of one pair of light strain transducers and one pair of piezoelectric accelerometers with a built-in amplifier. The two gauge pairs are normally bolted onto the pile about 0.5 m, or two to three pile diameters, below the pile head. The signals from the gauges are transmitted by a connector box hung below the pile head. From there, one cable carries the signals to the Analyser.

The Analyser, in receiving the signals from the gauges, will calculate and print out on paper-tape three values. The operator can select the three values from among several different alternative values, such as impact force, maximum force, developed energy, etc., and a computed estimate of the mobilized soil resistance (discussed below).

In computing the output values, the Analyser makes use of operator entered calibration factors of the gauges, pile related data, such as mass, length, wave speed, and other variables.

Simultaneously with the print-out provided by the Analyser, the oscilloscope displays the traces from the two gauge pairs. The primary use of the oscilloscope is to enable the operator to verify that the monitoring system functions properly. However, the visual display of the traces provides the operator with a valuable support for an on-the-spot judgement of the pile integrity, capacity, and general behaviour.

The measurements are, as mentioned, stored on a tape recorder. When playing back the tape through the Analyser, the original driving is simulated. Values, which were not selected for print-out the first time, can now be obtained in a new output mode.

An important additional advantage of storing the measurements on the magnetic tape is that the data can be processed in the laboratory by means of CAPWAP analysis (discussed below).

For additional information on the use of the Pile Driving Analyser System, see Gravare and Hermansson (1980), and Gravare et al. (1980).

#### Wave Traces

Fig. 8 shows an example of measured force and velocity resulting from a hammer blow, as wave traces drawn against time. The time scale is marked in  $L/c$ -units. As shown, the pile head first accelerates to a peak velocity,  $v$ , coinciding with the peak force in the pile. This peak defines the time of impact (more stringently, the time of impact is defined as time of zero acceleration). Thereafter, the magnitude of both the velocity and the force decreases. At time  $2L/c$  after the impact, the reflected wave from the pile tip is observed in the measurements (at the pile head).

When the reflected wave is a tension wave, the measured net force will decrease, while the net velocity will increase (upper diagram). When the pile encounters tip resistance, the reflected wave at time  $2L/c$  is a compression wave increasing the measured force, while decreasing the velocity of the pile (lower diagram).

One of the most useful aspects to consider in the visual study of the force and velocity wave-trace diagram is that an input of force, such as a hammer blow, a helmet bounce, etc. will show a parallel behaviour of the two traces. Reflections, however, whether in tension or compression, will have the opposite effect on the wave traces, i.e., separate them from each other.

When the velocity trace is plotted to the scale of velocity times impedance,  $v$  times  $EA/c$ , the two traces are proportional. Therefore, before any reflections have been superimposed, the traces plot on top of one another. Later, when, for instance, reflected compression from soil resistance along the pile shaft reaches the gauges at

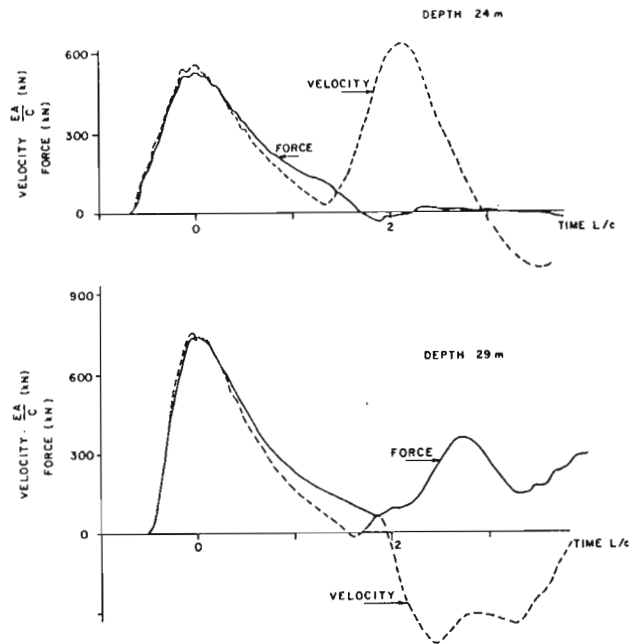


FIG. 8

Force and velocity traces from a precast concrete pile at easy driving against little end resistance (upper diagram) and at harder driving against more significant end resistance (lower diagram). ( $L = 43$  m,  $A = 0.150$  m<sup>2</sup>).

the pile head, the two traces separate. The location of the separation indicates where in the pile the shaft resistance occurs, and the extent of the separation is an indication of the magnitude of the shaft resistance. The traces in Fig. 8 illustrate the above showing how the traces initially superimpose each other and, then, separate due to reflected compression originating from soil resistance along the pile shaft.

As mentioned, a reflected tension wave decreases the net force in the pile. At the pile head, the net force can never be appreciably negative, i.e., net tension cannot occur at the pile head, because tension in the pile pulls the pile head down from the helmet and hammer creating a free end, where forces must be zero. Therefore, the force trace (from pile head measurement) cannot be indicative of large or small tension in the pile. Instead, the velocity trace is used for this purpose.

When the strain-wave encounters the location of damage in a pile, such as a crack, a loss of cross section, a reduced stiffness due to local buckling, etc., a tension wave reflection is sent back to the pile head superimposing the impact compression wave. This is manifested in the records as a small "blip" on the traces, where the force decreases momentarily and the velocity increases. This visual effect makes the dynamic monitoring a very efficient tool for discovering damage in driven piles both as to the extent of the damage and as to its location.

Naturally, the "blip" can be treated analytically. The resulting quantitative information is useful in judging the pile and can be related to specified limit values in a given case. For complete discussions and examples on the interaction of the two wave traces, see Rausche and Goble (1978), Goble et al. (1980), Authier and Fellenius (1980a and 1980b), and Likins (1981).

The combination of the two indirect measurements of force and velocity is unique for the Case-Goble system. The original innovation was simple, but ingenious. The two traces allow a fruitful combination of quantitative measurements and analysis with engineering judgement and experience. In fact, the Case-Goble system has in one leap vastly improved the understanding of the complexity that is pile driving, and allowed a quantitative and factual approach to the design and construction of piling projects.

#### Developed Energy

The energy developed in the pile by the hammer blow is the integral over time of the product of measured force and velocity. The developed energy reaches its maximum value, when the pile starts to rebound, i.e., velocity becomes negative, whereafter it decreases (energy is sent back to the hammer). The maximum value of the energy developed in the pile is defined as the energy delivered to the pile by the hammer.

The developed energy is not exclusively dependent on the hammer size, condition, fuel, losses in anvil and cushion, etc., but also on the response of the pile and the soil to the hammer impact. A "soft" pile and/or a soft soil cannot provide enough resistance to the impact for a large energy to develop.

#### The Case Method Estimate of Mobilized Soil Resistance

The Case-Goble system allows a field estimate to be made of the dynamically mobilized soil resistance. The assumptions behind the computation are as follow.

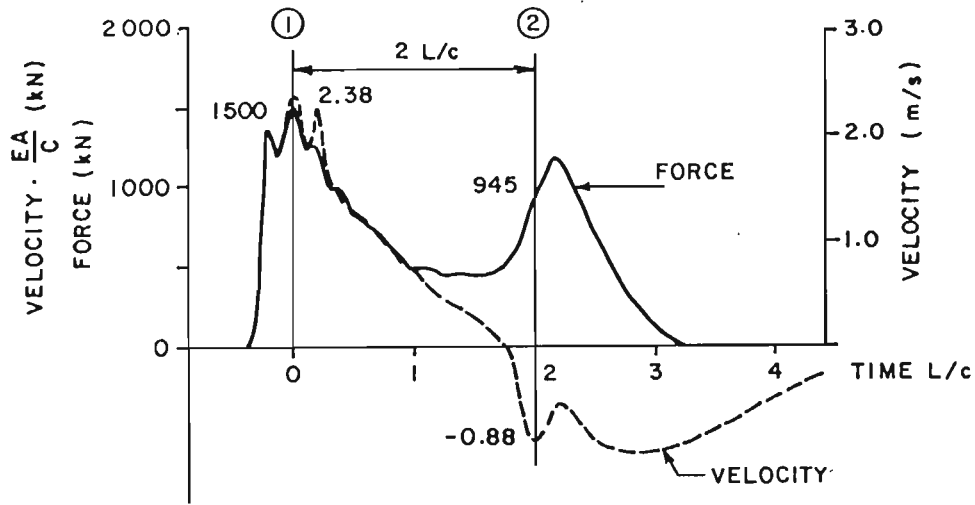
- \* the pile material is ideally elastic
- \* the pile is of uniform cross section
- \* the soil resistance along the pile shaft and at the pile tip shows rigid plastic behaviour

Based on the above assumptions and the theory of wave propagation in uniform rods, a mathematical relation has been established for the mobilized total (dynamic and static) soil resistance (Goble et al., 1970). In words, the relation simply says that the mobilized total resistance is the average of the force measured at the time of impact and at  $2L/c$  later plus the impedance of the pile times half the difference between the velocity values at impact and  $2L/c$  later.

In Fig. 9, an example is given of a computation of the Case Method Estimate of the mobilized total soil resistance from records taken when restriking a 34 m long precast concrete pile.

The mobilized total soil resistance is greater than the mobilized static soil resistance. (Note that the mobilized soil resistance is only equal to the ultimate soil resistance, if the pile has moved beyond the distance of the quake). The calculated static soil resistance, called Case Method Estimate, is the difference between the mobilized total resistance and the damping force. The latter is a function of the pile tip velocity, as computed from the measurements, multiplied with an input damping factor, J. Goble et al. (1970) have listed J-factors as empirically determined from correlation with static pile load tests.

The Case Method Estimate for determining the static soil resistance is considered



$$\begin{aligned}
 \text{MOBILIZED RESISTANCE} &= \frac{F_1 + F_2}{2} + \frac{EA}{C} \cdot \frac{1}{2} (V_1 - V_2) \\
 &\text{(dynamic and static)} \\
 &= \frac{1500 + 945}{2} + 638 \cdot \frac{1}{2} (2.38 + 0.88) \\
 &= 2260 \text{ kN}
 \end{aligned}$$

FIG. 9

Example of measured force and velocity as wave traces drawn against time, and calculation of mobilized total soil resistance according to the Case method (Precast concrete pile:  $L = 34 \text{ m}$ ,  $A = 0.080 \text{ m}^2$ ).

reliable, when used where previous studies and tests have proven the estimate to be correct for similar piles driven with similar equipment. When this is not the case, the method should be used conservatively, or preferably, be calibrated with a static test loading, as well as a complete laboratory computer analysis of the dynamic records by means of the CAPWAP analysis. (The CAPWAP analysis is discussed below). For detail information and discussion on the Case Method and variations on the approaches to correlate the computation with the field and to compensate for special conditions, see Likins and Rausche (1981).

The Case Method Estimate is sensitive to variation of the wave speed, because the time,  $2L/c$ , for a reflection from the pile tip to reach the gauge location is inversely proportional to the wave speed. In driven concrete piles, for example, the wave speed is affected by microcracks in the concrete developing in prolonged driving, slacks in splices, etc. Furthermore, the wave speed is a function of the concrete elastic modulus, which is not a constant, but can diminish with an increase of stress level in the pile (Fellenius, 1979).

The largest influence on the Case Method Estimate, however, originates from the quake distance. The assumption of rigid plastic behaviour of the soil means that the quake is assumed to be zero. This is, of course, not true, but it is an assumption commonly used in soil mechanics applications. The quake value is usually small, about 2.5 mm as mentioned above, although it can sometimes be much greater.

The strain-wave speed,  $c$ , in the pile varies from about 3000 m/s for wood to about 4000 m/s for concrete to about 5000 m/s for steel. As the pile tip has to accelerate and travel the distance of the quake before the peak of the strain wave is reflected, there is a delay of about a millisecond in the reflection of the peak force, which corresponds to a range of about 0.1 to 0.5  $L/c$ -units for most piles. The delay is neglected by the assumption of pure plasticity in the Case Method Estimate. For piles driven in low quake soils, and/or by diesel hammers, which have a significant wave rise-time, the assumption of ideal rigidity is usually of little consequence, however. Generally, unless the conditions are extreme, the value of the Case Method Estimate can reliably be correlated to actual static bearing capacity of a pile.

#### The CAPWAP Computer Analysis

The advantages of the Wave Equation Analysis and the field measurements by means of the Pile Driving Analyser have been combined in a computer program called CAPWAP (actually, a family of programs) developed by Case Western Reserve University, Cleveland. The CAPWAP analysis is very much superior to conventional Wave Equation Analysis, because in using input of actually measured data it is independent of both natural variations of input data and of subperforming hammers. The details of the method are given in a milestone paper by Rausche et al. (1972).

For analysis with the CAPWAP program, the measured analog force and acceleration curves are first digitized. Thereafter, the computer takes the acceleration curve and calculates with the aid of six operator-controlled variables a force curve, which is matched to the measured force curve. The six variables are, side and tip quake, side and tip damping, and load along the pile shaft and at the pile tip. The operator interacts with the computer, making several successive runs, each time improving on the match between the computed and the measured force curves. The results of the analysis is the distribution of the mobilized soil resistance, i.e., the ultimate static bearing capacity, when fully mobilized, and, also, the selection of variables used to achieve the final match.

Fig. 10 presents an example from four CAPWAP runs on data from a steel pipe pile driven with an air/steam hammer. The figure shows how, successively, the match is improved from the first trial to the fourth. Prior to time  $2L/c$ , the operator is essentially concerned with the shaft resistance modelling. At and beyond  $2L/c$ , the end resistance is included. Note that the effect of the hammer assembly drop at about  $5 L/c$  is matched, also. The match shown was achieved with an unusually small number of trials. Sometimes a good match requires more work. For examples of final force and velocity matches see Rausche et al. (1972), Rausche (1980), and Gravare (1980).

As in the case of an analysis of results from static test loading, the CAPWAP analysis is influenced by the effect of so called residual loads. The analysis assumes that all pile elements are unstressed before the analysed blow takes place. However, it is probable that often the previous blows have built in a stress between the soil and the pile precompressing the pile elements. This is illustrated in Fig. 11. The upper diagram shows an initial blow drawn assuming no precompression in the pile element. However, should the movement of the pile stop before all the forces are equalized, a precompression condition sets in resulting in a residual load and corresponding compression in the pile element. Then, for the next blow, as shown in the lower



diagram, when assuming that the pile element has no residual compression, the analysis (e.g. CAPWAP) will result in a larger than real soil resistance and quake values acting on the element. As the pile forces are in equilibrium, the overestimation will be compensated by underestimation of other elements, which are subjected to residual tension and, therefore, causing the analysis to result in smaller than real soil resistance and quake values. The total ultimate soil resistance will be unaffected, but not the distribution between the pile elements (i.e. of shaft resistance along the pile), and/or between shaft and end resistance.

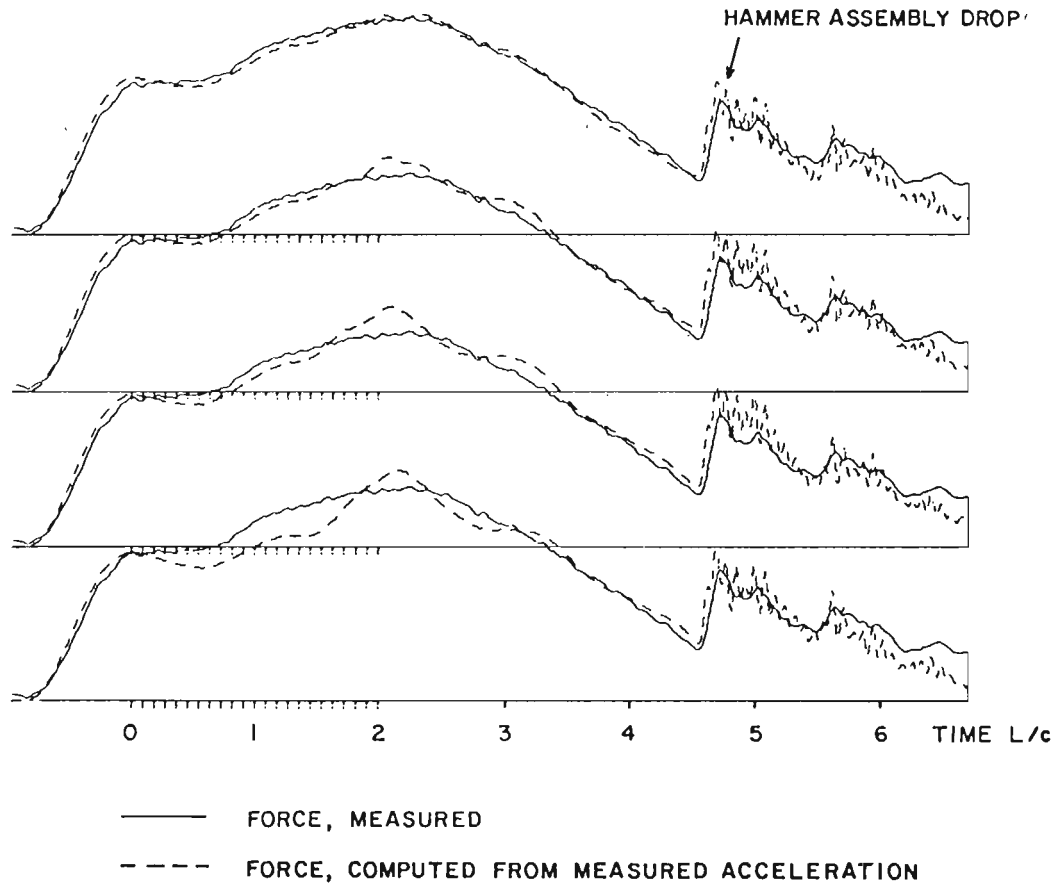


FIG. 10

Example of a successive CAPWAP force-match. Time for impact and the  $2L/c$  range are indicated. Note hammer assembly drop at about  $5L/c$ .

When required, the CAPWAP analysis can model the effect of residual loads. However, an acceptable match can be achieved by manipulating soil stiffness and damping, also. Therefore, residual loads and precompression, or pretension, cannot always be considered in a CAPWAP analysis. This is a minor issue, however, because during the driving of a pile, residual loads are rarely of any significant magnitude. They are, in fact, of much greater importance for the evaluation of results from static test loading. For additional views on this subject, see Holloway et al. (1978).

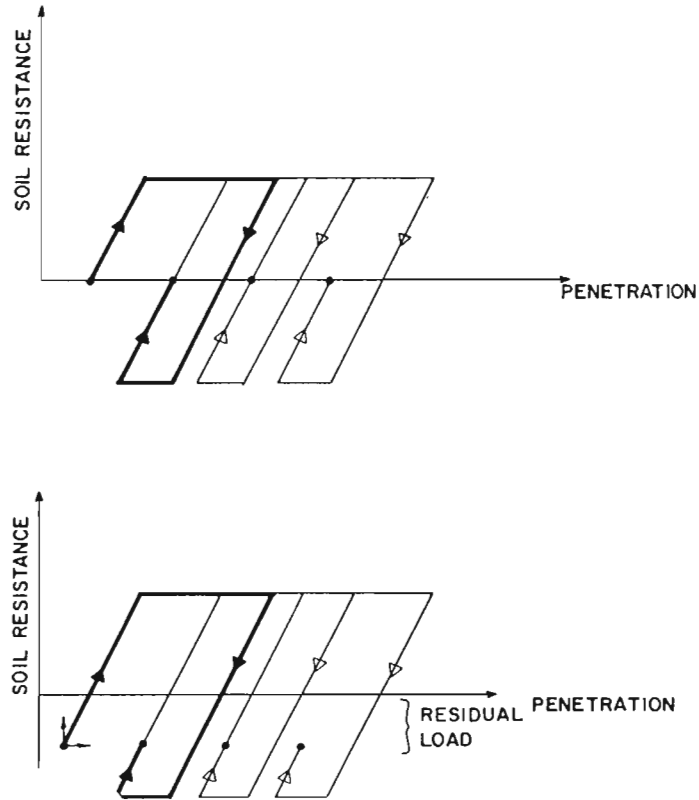


FIG. 11

Static soil resistance load-penetration diagram for a pile element.  
 Upper diagram: when unaffected by residual loads  
 Lower diagram: when affected by residual compression loads.

### Potential Applications of Dynamic Monitoring with the Pile Driving Analyser

The modern use of dynamic measurements in-situ paired with detailed analysis in the laboratory using the Pile Driving Analyser and CAPWAP analysis have removed much of the guesswork from pile engineering. Potential uses of the system are summarized below.

#### A. Hammer performance

- \* Measured energy versus manufacturer's rated value
- \* Effects of cushion properties and helmet assembly
- \* Effects of varied operating pressures, strokes, fuel changes, etc.
- \* Comparisons of different models and makes of hammer
- \* Hammer operating difficulties
- \* Whether the changes in blow-count are caused by soil changes or by (otherwise unknown) changes in the hammer performance

#### B. Pile performance

- \* Magnitude of driving stresses in the pile
- \* Extent and location of suspected pile damage
- \* Total length of existing piles

#### C. Pile bearing capacity

- \* Determination of mobilized soil resistance
- \* Location of adequate bearing strata
- \* Confirming or disproving adequacy of refusal criteria specified
- \* Providing data for CAPWAP and WEAP analyses

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